Abstract

Providing a high-quality mathematics education to young people in the US is of vital importance, and it is an area where there is a great deal of room for improvement. We suggest that one part of this process is a greater emphasis on applications of mathematics. At the high school level, a promising way to do this is by implementing an algebra course based upon applications to personal finance. This specific application has two key virtues. First, it is an authentic application that allows for sophisticated modeling activities that build conceptual understanding. Second, it is an application that is relevant to students, which builds motivation for engaging with the material.

Keywords: financial education; mathematics education; modelling; engagement
1. Introduction

Providing a high-quality mathematics education to young people in the US is of vital importance, and it is an area where there is a great deal of room for improvement. We suggest that one part of this process is a greater emphasis on applications of mathematics. At the high school level, a promising way to do this is by implementing an algebra course based upon applications to personal finance.

This specific application has two key virtues. First, it is an authentic application that allows for sophisticated modeling activities that build conceptual understanding. Second, it is an application that is relevant to students, which builds motivation for engaging with the material.

To make our case, we will begin by addressing the current levels of mathematics achievement in the US, and the importance of improving in this area. We will then summarize research showing that both authentic applications of mathematics and relevant applications improve student achievement. Finally, we will explain how a course applying high school algebra to personal finance meets these criteria.

Mathematics education in the US is currently in a worrying state, especially at the high school level. In an influential document, the National Council of Teachers of Mathematics (NCTM 2014) summarizes the situation:

- Average mathematics NAEP scores for 17-year-olds have been essentially flat since 1973. (NCES 2009)
- Only about 44 percent of U.S. high school graduates in 2013 were considered ready for college work in mathematics, as measured by ACT and SAT scores. (ACT 2013; College Board 2013c)
- Among cohorts of 15-year-olds from the 34 countries participating in the 2012 Programme for International Student Assessment (PISA), which measures students’ capacity to formulate, employ, and interpret mathematics… the U.S. cohort ranked 26th. (Organisation for Economic Co-operation and Development [OECD] 2013a)
- Only 16 percent of U.S. high school seniors are proficient in mathematics and interested in a STEM career. (U.S. Department of Education 2014). (NCTM 2014 p. 2)

Further, more recent research suggests the situation has not improved in the five years since this was written (ACT 2019).

Compounding the problem, research demonstrates that participating and succeeding in mathematics courses is one of the key predictors of financial success in the US today. As Arcidiacono puts it:
Students who choose natural science majors earn substantially more than humanities majors. In fact, economists have reported that differences in returns to majors are much larger than differences in returns to college quality. James et al. (1989) argue that “… while sending your child to Harvard appears to be a good investment, sending him to your local state university to major in Engineering, to take lots of math, and preferably to attain a high GPA, is an even better private investment.” (Arcidiacono 2004 p. 252).

To take advantage of this connection, students must come out of high school with a strong foundation in mathematics if they are to take on the kind of math intensive major that tends to lead to higher future earnings. Further research, from Goodman (2019), suggests, that just taking additional math courses in high school leads to increased financial benefits, regardless of whether this leads to a mathematics intensive major, especially for deprived demographic groups. He writes:

[E]ach additional year of math raised blacks’ earnings by 5-9%, accounting for a large fraction of the value of a year of schooling. The earnings impact of additional math coursework is robust to changes in empirical specification, is not driven by selection into the labor force, and persists when earnings are conditioned on educational attainment. The reforms close one fifth of the earnings gap between black and white males. (p. 2)

Further, Cole, Paulson and Shastry (2014, 2016) have found that additional mathematics courses increases a range of positive financial outcomes, not limited to increased income – for example, greater financial market participation, higher investment income, and better credit management. This should perhaps not be surprising, given that research shows that higher mathematics ability is correlated with better decision making in general (Peters et al 2006).
2. A Strategy for Improving Mathematic Education

Our focus will be on how to improve mathematics curriculum. There are many other elements to improving mathematics achievement: from additional teachers, to additional teacher training, to additional school resources, to anti-child-poverty initiatives. However, as a recent report by the Center for American Progress (CAP 2015) argues, implementing a high-quality curriculum is an efficient, cost-effective way to improve educational achievement.

One promising area for curriculum improvement is finding appropriate applications of mathematics. The benefits here are twofold: first it allows for effective mathematical modeling; second it increases student motivation to engage with the material. Below is an analysis of research, backing up these claims.

2.1 Mathematical Modeling

Developing skills in mathematical modeling is widely recognized to be an essential part of a high-quality mathematics education. It is a key point of emphasis in the Common Core State Standards for mathematics:

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace... Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. (National Governors Association 2010).

In addition, skills relating to modeling are intertwined with virtually all of the other ‘mathematical practices’ that should underpin all mathematics education, according to the Common Core standards.

A similar emphasis on modeling is found in the recommended ‘mathematics teaching practices’ of NCTM’s influential manifesto, Principles to Action:

An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world. (NCTM 2014 p. 70)
The mathematics curriculum should not only be coherent but also make connections from the mathematics curriculum to other disciplines. (NCTM 2014 p. 75)

Why is modeling so important in mathematics education? It encourages the kind of reflection and ‘deep strategy use’ necessary to build conceptual understanding, which is essential for lasting mathematics success (Bullmaster-Day 2006; Brown et al 2014). Further, as a matter of definition, any occasion in which students go on to use mathematics outside of the classroom will involve modeling.

When modeling, students can relate the math they’re learning to a tangible situation; this allows them to reflect on their calculations, in terms of what they say about the situation being modeled. It encourages in depth project-based learning, as students develop the math to model more aspects of the scenario and engage in experimentation as they alter the parameters of the model and observe its effects. Finally, they may be able to draw on their background knowledge of the scenario to assist in tackling the problem, as recommended by constructivist theories of education: they are able to relate what they learn mathematically to the application, which builds conceptual understanding (Ellis 2018).

In order for these benefits to be realized, however, the model must be an authentic application of mathematics, not a mere ‘word problem’. As Bonotto (2007) notes:

> Recent researchers have documented that the practice of word problem solving in school mathematics promotes in students the exclusion of realistic considerations and a "suspension" of sense-making, and rarely reaches the idea of mathematical modelling and mathematisation. (p. 185)

To highlight how word problems in mathematics are deficient, Gerofsky (1996) subjects them to linguistic analysis and finds that:

> During problem solving, school beginners concentrate on stories rather than on numbers; during interviews it has been found that such children often augment the story with what follows from their own knowledge or experience. Older children, however, always try to reach some solution, perhaps by a trial-and-error strategy … The claim that word problems are for practicing real-life problem solving skills is a weak one, considering that… unlike real-life situational problems, no extraneous information may be introduced. (p. 41)

The key point here is that in word problems the story is irrelevant, except insofar as it is the medium from which the mathematical components are to be extracted. Thinking carefully about the particulars
of the story, not only does not help in solving the problem, it can actually hinder it! Consider the following examples, also discussed by Gerofsky:

Every year Stella rents a craft table at a local fun fair and sells the sweaters she has been making all year at home. She has a deal for anyone who buys more than one sweater; she reduces the price of each additional sweater by 10% of the price of the previous sweater that the person bought. Elizabeth bought 5 sweaters and paid $45.93 for the fifth sweater. How much did the first sweater cost? (Ebos, Klassen, & Zolis 1990)

In this example, one must extract an equation to solve from the paragraph of text; however, the scenario described makes no sense if one tries to analyze it. The unknown quantity is the price of the first sweater, despite the fact that the description stipulates that one of the known quantities (the price of five sweaters) is determined by this unknown. There’s no possible way you could be in this situation, so it’s not practice for using math in real-world retail – trying to put yourself in Stella’s shoes would be no help in solving the problem, and could well lead to further confusion.

Or consider an example dating back to early medieval times:

A basin can be filled by three taps: the first fills it in sixteen hours, the second in twelve hours, and the third in eight hours. How long will it take to fill it when all are going together, if at the same time the basin is being drained by a pipe which can empty it in six hours? (Problem collected by Alcuin of York (circa 790 AD) as paraphrased in F P Sylvestre's Traiti d'arithmetique, Rouen 1818)

In this case again, one has to extract an equation from the text, and reflecting on the realities of medieval bath filling is of no help. Imagining a servant faced with the task of filling a bath for their master, the first step would be to plug the pipe that the bath is draining from – thus making the problem moot!

Such activities may be useful in developing linguistic reasoning, but they do not help with the higher level mathematical abilities that are required for modeling in the true sense of the world. As Masingla et al (1996) put it:

Knowing and using students' out-of-school mathematics practice is important in school situations because it provides contexts in which students can make connections. Connection making is essential in constructing mathematical knowledge but at present is often absent in classrooms. (p. 194)
In addition, Ellis notes that authentic modeling task can be used create what he calls “Culturally Responsive Teaching of Mathematics”:

Culturally responsive mathematics teachers leverage mathematical learning by expanding children’s mathematical thinking, building bridges between previous knowledge and new knowledge... fostering connections with cultural funds of knowledge and experiences, and cultivating critical mathematical knowledge that enables students to analyze and address authentic problems. (Ellis 2018 p. 6)

To summarize, effective use of mathematical modeling is one aspect of a high-quality mathematics education. This requires two key factors. First, the applications must be authentic rather than mere word problems, so that analyzing the scenario being modeled enhances the activity, and promotes learning. Second, the situation being modeled should be relevant to the students, so they can connect it with their existing knowledge and enhance their conceptual understanding.

We will show that applications to finance allow for exactly this kind of modeling. First, though, we will discuss another important element of improving math education: student engagement.

2.2 Student Engagement

One of the key factors in determining whether students succeed in mathematics is whether they are engaged with the subject matter. When students are engaged with material, they invest attention and effort in understanding topics and solving problems. Crucially, this is not about brute number of hours spent working, but about the type of activity the student performs: it must be the kind of high-level cognitive engagement which requires much effort and attention. As Greene puts it, engagement requires meaningful, as opposed to shallow, cognitive processing:

Strategies that involve meaningful (i.e., elaborative) processing attempt to connect or integrate new information with existing knowledge in an effort to form a richer, more coherent mental representation (Weinstein & Mayer, 1986). Shallow processing strategies (i.e., rote processing), such as underlining or mechanically rereading the new information, produce a less elaborate memory representation, limiting the retrieval and generalizability of the information. Research has consistently found that meaningful processing strategies lead to greater performance on achievement measures over the material studied than shallow strategies. (Greene 2004 p. 463)
Therefore, in order for students to effectively learn mathematics, they must engage in high level cognitive processing with the subject matter. This suggests that we should identify what factors promote these ‘meaningful processing strategies’. On this Greene writes: “Three motivational factors that have been consistently related to cognitive strategy use in learning situations are self-efficacy... achievement goals... and perceived instrumentality” (Greene 2004 p 463).

The first two factors are somewhat removed from choice of curriculum. ‘Self-efficacy’ refers to the idea that students are more engaged when they have belief in their own ability to be successful in the task. ‘Achievement goals’ refers to what the student is aiming for when completing the task: they are more successful when they are concerned with mastering the subject matter, an ‘achievement goal’, than when they are concerned with getting a good grade, a ‘performance goal’.

The factor that can be most directly affected by choice of curriculum is the third: ‘perceived instrumentality’, which Greene (2004) describes as ‘the extent to which school tasks are perceived as instrumental to attaining personally valued future goals’. For a subject, or topic, to have high instrumental value for a student: they must believe that developing skills and knowledge in that area will allow them to meet important goals down the line.

Why is instrumentality important? First, based on a theoretical account of human decision making: expected value theory. In order to decide to engage with the study material, students need to believe putting in the effort will be worth their while. Therefore, the student must decide whether the benefits they will achieve if they are successful in learning, weighted according to how likely they are to succeed, outweigh the costs associated with making the effort. If it does, they will decide to try; if it does not, they will not. (See Wigfield, 1994)

Of course, this is an idealized model of how a completely rational student would decide how hard to work, which is not a perfect match with reality. As Pintrinch and De Groot (1990) note, there is an emotional component to a student’s willingness to engage.

Despite this, there is strong evidence that increasing perceived instrumentality increases student engagement – even if they don’t always act in perfect accordance with expected value theory. As Wigfield (1994) notes: “Students’ valuing of mathematics most strongly predicts their intentions to continue taking mathematics courses and their actual decisions to enroll in advanced math during high school.” (p. 54) Similarly, Miller (1999) writes:
Consistent with the theoretical claim, regression analyses indicated that perceived instrumentality was a significant predictor of both intrinsic and extrinsic valuing... These findings support the important role played by students' perceptions of the connection between academic tasks and their valued future goals, and suggest that facilitating perceptions of the instrumentality of schoolwork may be critical to fostering increased proximal motivation for academics.

Further, increasing perceived instrumentality of mathematics is especially important at the high school level, as this is the time engagement tends to decline. As Wigfield (1994) notes:

Early adolescents' beliefs and values tend to become more negative in different subject areas following the transition to middle school. During adolescence, beliefs about certain school subjects, especially math, continue to become more negative.

Changing the subject matter of high school mathematics to topics a student values is a promising way to reverse this trend of declining engagement. Though the focus here is only on one of three factors promoting engagement, it’s plausible to think these factors are mutually reinforcing. If the student has future goals they are focused on, then what counts for them is mastery rather than performance. In addition, if the initial motivation leads the students to increased success, that will in turn increase self-efficacy.

To conclude, in this section we have shown that mathematics courses should aim to increase student engagement by increasing its perceived instrumentality. In the next section we’ll explain why an application to finance meets this goal.
3. The Value of Personal Finance

Basing a high-school math course around applications to personal finance will allow for high-quality modeling activities and increase the perceived instrumentality of math for many students.

3.1 Modeling Finance with Mathematics

To understand the connection between these subjects, it’s necessary to take a step back and think about the nature of personal finance itself. The central topics in this area build upon two key principles:

- Transferring wealth across time
- Managing risk

This is a result of Modigliani’s Life Cycle Hypothesis (Ando & Modigliani 1963). In order to meet our needs at all points in our life, we need to transfer wealth acquired in periods when income is high (i.e. the peak of one’s career) to periods when income is low (i.e. as a student and after retirement). Financial tools allow us to do this. Many familiar tasks in personal finance fall under this principle: from budgeting and saving to borrowing and investing.

Crucially, transferring wealth almost always involves interest: you pay it when you borrow, and you earn it when you invest. With interest comes the mathematics of exponents, logarithms and geometric series. Modeling, for example, retirement savings provides a complex and authentic application of these mathematical topics (See Marley-Payne & Dituri 2019).

With regard to the second principle, at various points in our lives we will face financial risks: for example, the possibility we will need an expensive medical procedure or that our home will be destroyed in a fire. If the risk is higher than we can tolerate, we can use financial tools to mitigate it – we can pay someone else to take on the risk for us, by purchasing insurance. In addition, if we are able to take on additional financial risk, we can receive payment for doing so – for example by investing in the stock market.

As a matter of definition, any scenario concerning risk will involve probability. Further, when assessing investment decisions, one must draw upon expected value and statistical topics, such as normal distributions. Again, this provides ground for high-quality, authentic modeling exercises.
As we show in detail in a companion paper, one can build a rigorous set of standards for a mathematics course built around these ideas, which has a series of inter-related modeling activities of authentic applications to personal finance running through it (Marley-Payne, Dituri & Davidson 2019).

3.2 The Perceived Instrumentality of Finance

The reason why applying math to personal finance increases perceived instrumentality is intuitively clear. Students will need to make good financial decisions throughout their lives, and improving their financial knowledge will help them do so. This is where the fact the applications are authentic, as discussed above, is essential – the math they are learning really will help with financial problems they are likely to face in the future.

In addition, research shows that current high-school students are in fact concerned about their financial future – showing that this is a goal they will personally endorse. A number of studies have found that high school students are concerned about their finances and interested in learning more (H&R Block 2016; NPR 2017; Ohio State News 2015).

Given that this is the case, there is good reason to think that focusing on financial applications of math will increase perceived instrumentality which will in turn increase student engagement with the subject matter. Based on this, we created a study to confirm this directly.

We gave a set of questions to 250 students participating in a math and finance program at the start of the 2019-2020 school-year (before they had taken the course). In this survey, this we provided a list of common applications of mathematics, including personal finance. We asked students to rate each subject on a scale of 1-5 both in terms of their interest in the subject matter, and how important they thought it was to learn about the subject. These questions assessed the intrinsic value and instrumental value of the subjects, respectively.
These results tell us a couple of interesting things. In line with our hypothesis, and validating our approach, students on average ranked personal finance highest, out of the given options, for both interest level and importance. In addition, it is worth noticing that the ranking of subjects below personal finance changed between the two questions. Most notably, sport was ranked a close second in terms of interest, but a distance fourth in terms of importance – and roughly the converse for
economics. This demonstrates that students were distinguishing the question of interest and importance, emphasizing the significance of personal finance ranking first in both.

Taken together, this suggests that integrating financial applications into a mathematics course will improve mathematics education by increasing engagement.
5. Conclusion

We have argued that basing mathematics coursework around applications to personal finance is a promising avenue for improving mathematics achievement. The motivation so far has been primarily theoretical, and leaves room for promising future research projects. First, we need a more rigorous study of how students value finance, in comparison with other applications of math. In addition, we should look at how connecting math with finance affects perceived instrumentality of math and look at how it affects math achievement.
References


About Us

As a not-for-profit organization that is passionate about personal finance and mathematics education, we created a high school course that embeds financial topics into a robust math course, Financial Life Cycle Mathematics. Our research shows that a math-based approach is the more effective way to teach financial literacy as it gives an in-depth, mathematically grounded understanding of personal finance topics.

About The FiCycle Course

Financial Life Cycle Mathematics (FiCycle Mathematics) provides high school students with an understanding of essential personal finance topics and the associated mathematical tools. FiCycle Mathematics is a project based curriculum that allows students to delve into real-world problems related to the themes of each unit. The math of the course is roughly at the level of Algebra II. We are confident that teaching these financial concepts in a mathematics course will provide students with the complete toolbox to make fully informed financial decisions.

To learn more about the course, what we offer, and how the course can fit into your school’s math sequence, visit https://ficycle.org or email info@ficycle.org.

Thank you to all of our generous donors who make it possible to serve the students enrolled in the FiCycle course!

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