# FiCycle Standards for Personal Finance and Mathematics

## 1. Our Principles

At Financial Life Cycle Education (FiCycle) We believe it is essential for students to receive a high-quality education in both personal finance and mathematics. Further, we believe that offering students a mathematics course that is based upon applications to finance advances both goals. In this document, we present the research and theory behind these beliefs and outline a set of standards for a course in personal finance and mathematics.

Our research-based approach rests on the following principles.

1. Students learn best when they develop conceptual understanding.
2. A conceptual understanding of personal finance requires connecting it to the underlying mathematics.
3. Students learn best when they find the material interesting and relevant to their lives.
4. Many students who are disengaged with pure mathematics find applications to finance relevant and interesting.
5. Student and teacher time is a scarce and valuable resource.

Principles 1-2 explain why we believe that teaching finance within a mathematics course improves the quality of finance education. Principles 3-4 explain why we believe basing a mathematics course around applications to finance improves the quality of mathematics education. Principle 5 is in effect a cautionary note that guides our thinking: it says that introducing a new course into the curriculum can come with a huge opportunity cost, since another course must be removed from the students schedule to make room. We believe that replacing a traditional third-year or fourth-year mathematics course with a mathematical finance course is uniquely positioned to justify this cost, as students in this case retain the additional mathematics training while gaining additional benefits through the financial applications.

In what follows, we outline the justification for these principles.

## 2. Understanding Personal Finance

Research shows that young people across the US are in a precarious financial situation, and many lack essential financial knowledge.[1] Further, it is known that this low financial knowledge is closely connected to undesirable financial outcomes.[2]

Financial education has been proposed as a solution to this pressing problem. Though such education programs have been shown, on the whole, to improve financial knowledge, their

effectiveness on financial outcomes varies greatly. It is important, therefore, to better understand the best practices that lead to effective education programs.[3]

A key finding in education research is that effective learning requires conceptual understanding. Being taught a series of procedures for completing a list of rote tasks, without addressing the connections between them, leads students to learn little, and struggle to remember what they do learn. Students learn and retain knowledge much better when they grasp the fundamental concepts connecting the different areas.[4] Since one of the key goals of financial education is for students to retain what they learn and apply it to key financial decisions in adulthood, long term knowledge retention is particularly important for financial education.[5]

Further, students will have to apply their financial knowledge in a complex and rapidly changing financial environment. By the time they reach adulthood, the financial products available and related terminology will likely be radically different from when they were in school. The decisions they face may involve difficult trade-offs with strong emotional components as well as potentially misleading marketing from financial providers. Being able to critically evaluate financial options in such scenarios based on learning in school will require an especially deep and robust level of conceptual understanding and knowledge of financial vocabulary will not be sufficient.

Thus, a high-quality education in finance should focus on developing conceptual understanding. Consistent with academic financial theory, the central concept in personal finance is the financial life cycle.[6] Therefore, financial education should include instruction in:

- How and why individuals and households transfer consumption over time.
- What financial risks individuals and households face and how they manage those risks.

These concepts are reflected in four essential understandings:

F1. The fundamental measure of financial wellbeing is wealth; financial statements are used to measure wealth over the financial life cycle.

F2. The transfer of consumption forward and backward over time is generally financed via payment series with compound interest. Present value and future value are the tools used to evaluate payment series.

F3. Risk can be measured using probability and expected utility. These tools provide the means to evaluate risk management tools such as insurance.

F4. Investments in the stock market can be modeled using probability distributions to assess risk and return. These tools can be used to demonstrate that for sufficiently diversified stock

market portfolios, over a long enough time, the additional expected return is likely to exceed the incremental risk relative to other investments.

Understanding these principles requires grasping a range of mathematical concepts and relating them to the relevant financial principles. Each of these financial understandings is directly related to a corresponding mathematical understanding, and the two should be used in concert with one another, as the course combines math and finance at a fundamental level. The standards are designed so that the course can be viewed either from the perspective of a financial education course that utilizes mathematics or a mathematics course that utilizes applications to finance.

Mathematical modeling is the process by which real-world phenomena are represented by mathematical relationships that can be expressed with numbers and equations. While mathematical modeling is often viewed as an independent standard in high school mathematics, in this course modeling is integrated into all of these standards as they require "using mathematical tools and methods to ask and answer questions about real-world situations," using modeling both as a vehicle for learning mathematics and as content for students for students to enhance their skills as modelers. [7]

M1. The dynamics of financial transactions and wealth can be modeled using financial statements, which are mathematical models that utilize concepts of algebraic manipulation and linear relationships to capture the relationships between income, expense, assets, and liabilities.

M2. Time can be modeled mathematically using natural numbers, integers, rational numbers, and real numbers. The mathematics of time involves series, sequences, limits, exponents, logarithms, and other functional forms. In particular, the mathematics of compound interest involves exponential functions, their inverses, and geometric series.[8]

M3. The mathematics of financial risk can be modeled with random variables. Random variables represent a combination of probability and outcomes and are often evaluated using expected value and other measures.

M4. Investment outcomes can be modeled with probability distributions, such as the binomial distribution and the normal distribution. The movement of stock prices is often modeled as a sequence of random variables sometimes called "random walks."[9] Generally, the standard deviation of such processes increases with the square root of time, while returns increase exponentially.

**3. Making Mathematics Relevant**

Math achievement levels in the US are lower than would be hoped, especially at the high school level. US students consistently rank poorly compared to those in other developed countries.[10]

Further, research demonstrates a strong link between mathematics achievements and future outcomes. In particular, high levels of mathematics education are associated with high income levels.[11]

A significant problem is student disengagement. In math class in particular, it's often reported that students are uninterested in the materials, and don't view it as related to 'real life'. It has been shown however, that learning is much more effective when students are highly engaged. One way to improve engagement is through the material being taught. Studies show improved results when students find the material relevant and interesting.[12]

Mathematics leaves much room for improvement in this regard. Many so-called real-world applications of math are contrived word problems, where the supposed application is irrelevant to the solution of the problem. What's needed are authentic applications, where the mathematical processes can be used to guide 'real life' decision-making.

We believe that applications to finance are well-suited for this purpose. High school level mathematics is required to understand a range of financial problems that students will likely have to deal with in the future, making for an authentic application. In addition, students are interested in finance so they will find this work interesting and relevant.[13]

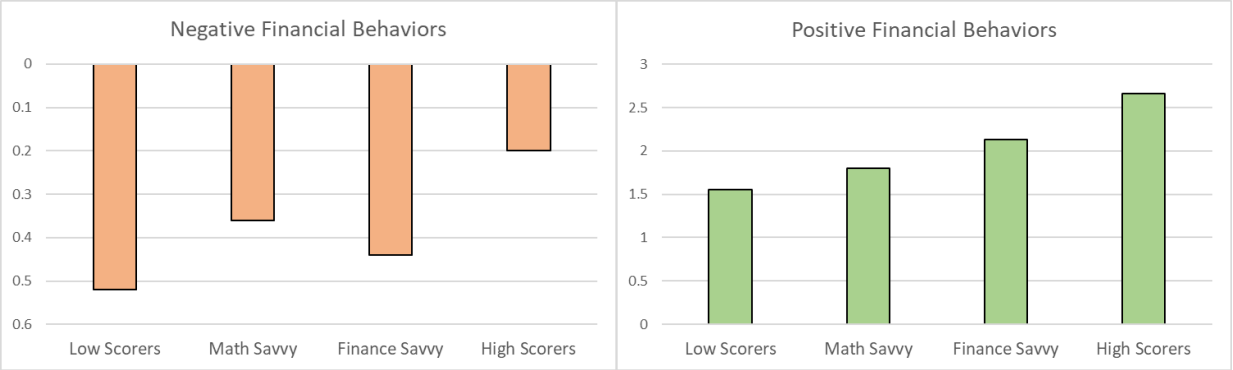
Moreover, many of the topics covered in K-12 mathematics were first developed or inspired by financial problems. This includes negative numbers, first introduced as a debt; concepts of equality featured in financial statements; limits associated with continuous compounding; and probability and statistics related to annuities.[14] Teaching these mathematical concepts in the context of finance returns them to their original development.

Integrating financial applications into a mathematics course will improve mathematics education by increasing engagement. The connection between mathematics and finance education is bi-directional, as a financial context helps students grasp and retain principles of mathematics, and mathematics skills enhance financial decision making.

**4. Financial Decision Making**

Research we conducted with the FINRA Foundation and the UChicago NORC indicates that people with both math skills and financial skills significantly outperform those with financial knowledge but not math skills.[15] Generally, the research shows that people with financial education have greater financial confidence and therefore engage in more financial activities. Unfortunately, these people engage in both positive and negative financial activities. Math is a differentiator, whereby negative actions are reduced and positive actions are retained and even increased.

Association Between Knowledge and Total Financial Actions Taken)[16]




**5. Opportunity Cost**

Our fifth principle states: "Student and teacher time is a scarce and valuable resource." There are only so many hours in the school year, both for students to learn, and for teachers to teach. And these hours are all, already, in use. Therefore, adding something to the curriculum requires removing something else. To justify such an addition, it must be shown that the benefits of adding the new course outweigh the costs of removing the old one.

This is a significant hurdle for implementing a standalone non-math-based-course in personal finance. To do this, a course in an unrelated subject matter must be eliminated – from the arts, sciences, or humanities. For example, a financial education initiative in North Carolina required cutting the high school American history coursework in half. Arguing that the benefits here clearly outweigh the costs is a tough case to make.

We would also caution against the "no cost option" of making finance education an extra-curricular course. Though this appears to offer the benefits of financial education without any cost, since nothing has to be cut, the appearance is deceptive. First, of course, students taking one extra-curricular activity prevents them from taking another – and there are all kinds of benefits to students participating in sports or creative arts, among other things. Second, an extra-curricular course is likely to be shorter and less comprehensive, which research shows is less effective.[17]

A math-based financial education course, on the other hand, does not require eliminating a course, but instead changing the focus of one high school mathematics course - for example a traditional Algebra 2 course or a fourth-year mathematics elective. The opportunity costs here concern any learning opportunities that are present in such a math course that are not in a financial mathematics course. And there certainly will be differences, for example, the sequence of mathematical topics in the standards presented below differ from in a traditional algebra course in that these standards have less of a focus on polynomials and more of a focus on modeling and probability. In addition, it should be noted that a course in financial mathematics

will probably cover fewer mathematical topics in total than a traditional one, since time must also be set aside to discuss the financial concepts and applications.

We do not believe these costs are severe, based on the previous discussion of best practices in mathematics education. Beyond mastering arithmetic, the significance of mathematics education lies primarily in developing mathematical *reasoning*, rather than getting students acquainted with as many mathematical concepts as possible: quality rather than quantity is the goal. Further, as spending time exploring mathematical applications to finance increases students' engagement with and understanding of mathematical concepts, the tradeoff is justified.

The mathematics topics covered by our essential understanding are also particularly well suited to prepare students for college level mathematics. A firm understanding of sequences and series, the idea of manipulating equations as a matter of identity rather than 'solving for x' and interpreting the input of a function as time are all crucial preparation for calculus. A fundamental understanding of random variables and probability distributions is an essential foundation for the math required in social science. Further, a combination of these elements is needed for stochastic calculus, which is required for a range of career paths in science, finance, and tech.

**6. Unpacking the Principles**

As we discussed in section 2, we believe that a high-quality personal financial education should provide understanding of the financial and mathematical concepts necessary to navigate the financial life cycle.

For a person to achieve good financial outcomes, they must have the financial means to meet their goals across their whole lifetime. One's financial needs and abilities vary over the course of one's lifetime, and one must plan for this when making financial decisions; this is what we mean by 'navigating the financial life cycle'. To do this requires understanding of:

*How and why individuals and households transfer consumption over time.*

- One's income varies across one's lifetime, as do one's necessary expenses, but they do not always line up, so one must use financial products to make sure one has the necessary resources available at different points in time to meet these needs.

*What financial risks individuals and households face and how they manage those risks.*

- One's future, financially speaking, is uncertain, and some possible outcomes would be undesirable if sufficient financial resources were unavailable. Financial products can be used to reduce the risk that the necessary resources will not be available.

As we discussed, fully grasping these ideas requires four essential financial understandings and the associated mathematical understandings.

F1. The fundamental measure of financial wellbeing is wealth; financial statements are used to measure wealth over the financial life cycle.

- To think about navigating the financial life cycle, we need to know how to adequately measure a financial situation. This requires thinking in terms of wealth, rather than cash balance. Students must understand that their wealth is the means by which they can meet their needs across the life cycle.

F2. The transfer of consumption forward and backward over time is generally financed via payment series with compound interest. Present value and future value are the tools used to evaluate payment series.

- Borrowing and investing involves compound interest, which makes understanding how to transfer consumption inseparable from a mastery of the mathematics of exponential functions and geometric series. Further, in real-life instances of transferring consumption, payments frequently will be made at regular intervals, rather than in lump sum. That means students must move beyond the simple calculations of compound interest that are typically taught, and work with interest within geometric series.

F3. Risk can be measured using probability and expected utility. These tools provide the means to evaluate risk management tools such as insurance.

- The fundamental concept when making any kind of decision under uncertainty is expected utility – and this includes financial decisions. To understand this, students must master the mathematics of probability and expected value. Combining the mathematical tools with a critical analysis of uncertain financial situations allows them to properly understand the importance of insurance.

F4: Investments in the stock market can be modeled using probability distributions to assess risk and return. These tools can be used to demonstrate that for sufficiently diversified stock market portfolios, over a long enough time, the additional expected return is likely to exceed the incremental risk versus other investments.

- Building significant financial wealth in the long term generally requires investing in the stock market, so it is essential for students to understand how to approach this in an informed manner.

## 7. The Standards

These are the FiCycle Standards for a course in personal finance and mathematics. They include both finance and mathematics standards and grouped according to essential understandings.

<table>
<tr><td colspan="2">Finance F1. The fundamental measure of financial wellbeing is wealth; financial statements are used to measure wealth over the financial life cycle.<br>Mathematics M1. The dynamics of financial transactions and wealth can be modeled using financial statements which are mathematical models that utilize concepts of algebraic manipulation and linear relationships to capture the relationships between income, expense, assets, and liabilities.</td></tr>
<tr><td>Finance Standards</td><td>Mathematics Standards</td></tr>
<tr><td>F1.1 Students know that wealth is defined as ability for consumption and understand its significance over and above cash balance.</td><td>M1.1 Students are able to manipulate and substitute linear equations, including key financial applications:<br>a. The net worth equation: $NW = A - L$ (net worth = assets - liabilities.<br>b. The net income equation: $NI = I - E$ (net income = gross income – expenses)</td></tr>
<tr><td>F1.2 Students understand the four key concepts for measuring wealth: income, expense, asset and liability.<br>a. Students are able to track and calculate these values using financial statements: income statements, balance sheets, and budgeting table.</td><td>M1.2 Students understand the relationship between an equation and a function taking elements of the equation as inputs. They apply this when relating financial equations to financial statements. (For example, they create a function that gives net worth as an output using the asset and liabilities entries on a balance sheet as inputs.)</td></tr>
<tr><td rowspan="2">F1.3 Students understand the key factors that influence wealth:<br>a. The connections between income level, career, and education.<br>b. The difference between incurring an expense and purchasing an asset.<br>c. The importance of maintaining a cash balance to preserve liquidity for emergencies.<br>d. The relationship between taxes and income.</td><td>M1.3 Students are comfortable with units and percentages, when dealing with currency and taxes respectively.</td></tr>
<tr><td>M1.4 (+) Students understand piecewise functions and use them to model total tax, marginal tax rate, and effective tax rate.</td></tr>
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<tr><td colspan="2">Finance F2. The transfer of consumption forward and backward over time is generally financed via payment series with compound interest. Present value and future value are the tools used to evaluate payment series.<br>Mathematics M2. Time can be modeled mathematically using natural numbers, integers, rational numbers, and real numbers. The mathematics of time involves series, sequences, limits, exponents, logarithms, and other functional forms. In particular, the mathematics of compound interest involves exponential functions, their inverses, and geometric series.</td></tr>
<tr><td>F2.1 Students understand the role of borrowing and investing in transferring consumption across the financial life cycle.<br>a. If one has a surplus of income now, one can invest it in order to use it at a future time when one's consumption needs are higher than one's income.<br>b. If one's current expenses exceed one's current income, one can borrow money to meet them, and repay the money with future surplus income.</td><td>M2.1 Students understand the rules of exponents, including negative exponents, and are comfortable manipulating them in algebraic expressions.<br>a. Understand and employ the compound interest formula<br>$$FV = PV \bullet (1 + \frac{r}{n})^{n \cdot t}$$<br>b. Understand and use the discounting formula<br>$$PV = FV \bullet (1 + \frac{r}{n})^{-n \cdot t}$$</td></tr>
<tr><td>F2.2 Students understand that the value of money changes over time due to interest: a dollar today is worth more than a dollar in the future.<br>a. Interest is earned or paid as a percentage of the value being transferred.<br>b. The connection between present value and future value is calculated using the compound interest equation.</td><td>M2.2 Students understand Euler's number, and use it in the continuous compounding formula<br>$$FV = PV \cdot e^{rt}$$<br>a. (+) Students understand the definition of Euler's number as a limit and use this to derive the continuous compounding formula.</td></tr>
<tr><td>F2.3 Students know the different borrowing needs one may face and the financial instruments for meeting those needs.<br>a. Buying a house builds wealth, through spending money on housing equity, an asset, rather than rent, an expense.<br>b. A mortgage is loan used for buying a house.<br>c. Spending money on college gives one a qualification that can lead to a future career with a higher income.</td><td>M2.3 Students the importance of estimation and how to employ it effectively. Students apply this to using the rule of 72 to estimate the time it takes for an investment of debt to double in value.<br>$$t = \frac{72}{R \cdot 100}$$<br>a. Students understand the rules for logarithms, including natural logs, and use this to derive the rule of 72.</td></tr>
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| d. Student loans are used to cover the costs of education.<br>e. A credit card can be used to borrow small amounts of money instantly, but comes with a high interest rate.<br>f. Credit scores determine one's access to a variety of borrowing opportunities, and are determined by past financial behavior. | |
| F2.4 Students know the different needs for investing one may have and the financial instruments for meeting those needs.<br>a. Savings accounts allow one to invest money with very low risk, high flexibility, and low interest rates.<br>b. Treasury Bonds are a tool for borrowing with low risk, low flexibility, and modest interest rates.<br>c. Stocks are higher risk investments with higher returns on average.<br>d. Getting a mortgage requires making a down payment, which one will have to save money for.<br>e. Upon retiring, one will no longer have income to meet one's living expenses, so one must invest for this while working.<br>f. One may have to deal with unexpected expenses which one should save in preparation for. | M2.4 Students understand how to calculate and model with arithmetic and geometric series and apply them to payment series appropriately.<br>a. Series of simple interest payments can be modeled with an arithmetic series.<br>b. Series of compound interest payments can be modeled with a geometric series.<br>c. Series can be created using recursive and explicit formulas for sequences.<br>d. When modeling realistic examples, one must add additional variables for growth rate and inflation. |
| F2.5 Students know that investments and repayments typically occur through a series of payments over an extended period of time, and understand how this affects financial decisions. They can understand situations involving the following features:<br>a. Annuities and amortization<br>b. Growing Payment series<br>c. Inflation | M2.5 Students understand how to breakdown complex formulas into simpler constituents. They can apply this to complex payment series formulas. |

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<tr><td colspan="2">Finance F3. Risk can be measured using probability and expected utility. These tools provide the means to evaluate risk management tools such as insurance.<br>Mathematics M3: The mathematics of financial risk can be modeled with random variables. Random variables represent a combination of probability and outcomes and are often evaluated using expected value and other measures.</td></tr>
<tr><td>F3.1 Students understand the different kinds of financial risk one faces over the financial life cycle:<br>a. Healthcare costs<br>b. Vehicle damage<br>c. Property damage/theft<br>d. Device damage/malfunction<br>e. Dependent impoverishment due to death of care giver</td><td>M3.1 Students understand the fundamental features of probability and use this to measure financial risk.<br>a. $P(Sample\ Space) = 1$<br>b. $P(E) = \frac{\#(Outcomes\ in\ which\ E\ occurs)}{\#(Outcomes\ in\ the\ experiment)}$<br>c. If $a$ and $b$ are independent events, then $P(a\&b) = P(a) \cdot P(b)$</td></tr>
<tr><td>F3.2 Students understand how insurance mitigates risk, and how the nature of the risk affects the need to purchase insurance.<br>a. With insurance, you pay someone else to take on risk for you.<br>b. Diversified vs correlated risk<br>c. Catastrophic risk, and ability to cover with savings.</td><td>M3.2 Students understand the concepts of expected value and expected utility and apply them to financial decisions.<br>a. Expected Value Formula: $EV = \sum_{i=1}^{n} P(o_i) \cdot V(o_i)$<br>b. Expected Utility Formula: $EU = \sum_{i=1}^{n} P(o_i) \cdot U(o_i)$</td></tr>
<tr><td>F3.3 Students know about the different kinds of insurance and their distinguishing features.<br>a. Premium, deductible, copay, limit<br>b. Health insurance<br>c. Car insurance<br>d. Home insurance<br>e. Device insurance<br>f. Life insurance</td><td>M3.3 Students understand binomials distributions and calculate outcomes probability in binomial experiments. They are able to use this to model situations with diversified risk.<br>a. Binomial Theorem: $P\ (k\ sucesses) = \binom{n}{k} p^k (1-p)^{n-k}$<br>b. Binomial experiments can be represented by binomial trees.</td></tr>
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| *Finance F4: Investments in the stock market can be modeled using probability distributions to assess risk and return. These tools can be used to demonstrate that for sufficiently diversified stock market portfolios, over a long enough time, the additional expected return is likely to exceed the incremental risk versus other investments.*<br>*Mathematics M4: Investments in the stock market can be modeled using probability distributions to assess risk and return. These tools can be used to demonstrate that for sufficiently diversified stock market portfolios, over a long enough time, the additional expected return is likely to exceed the incremental risk versus other investments.* | |
| F4.1 Students understand the nature of investments in equities, and the specific concepts of shares, dividends and returns. | M4.1 Students can model stocks as random walks and understand how this is a form of binomial distribution. |
| F4.2 Students understand the Efficient Market Hypothesis, and the implications it has for investment decisions. | M4.2 Students understand the concepts of mean and standard deviation and can relate this to financial risk. |
| F4.3 Students understand the relationship between risk and return: that investors are generally risk adverse, and so higher risk investments tend to come with a higher expected return. | M4.3 Students understand the key features of the normal distribution, and the situations in which the binomial distribution approximates the normal distribution. |
| F4.4 Students relate their knowledge of stocks and risk to their understanding of the financial life cycle in order to evaluate when investing in stocks is and is not appropriate. | M4.4 Students can mathematically model investments with different portfolio sizes over different lengths of time and observe general patterns between these factors and financial risk. |

The Plus (+) Standards offer additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.

Notes:

[1] See, Hasler, A., Lusardi, A., & Oggero, N. (2018). Financial fragility in the US: Evidence and implications. Global Financial Literacy Excellence Center, The George Washington University School of Business. https://gflec.org/wp-content/ uploads/2018/04/Financial-Fragility-Research-Paper-04-16-2018-Final.pdf

[2] See Lusardi, A., & Mitchell, O. S. (2014). The economic importance of financial literacy: Theory and evidence. Journal of Economic Literature, 52(1), 5-44. https://doi.org/10.1257/jel.52.1.5

[3] We discuss this further in our academic literature review FiCycle 2019 "Combining Mathematics and Financial Education". (https://ficycle.org/wp-content/uploads/2022/02/FiCycle-Research-Overview-2022.pdf )

[4] See, Bullmaster Day (2006) A Synthesis of Research on Effective Mathematics Instruction. Touro College Graduate School of Education. https://www.academia.edu/7818450/A_Synthesis_of_Research_on_Effective_Mathemati cs_Instruction

[5] We discuss the importance of conceptual understanding in financial education in depth in our working paper FiCycle (2018) "Financial Education and Conceptual Understanding: Learning from Best Practices in Mathematics" https://ficycle.org/wp-content/uploads/2021/12/conceptual-understanding.pdf

[6] Ando, A., & Modigliani, F. (1963). The" life cycle" hypothesis of saving: Aggregate implications and tests. The American economic review, 53(1), 55-84.

[7] Appendix B: Mathematical Modeling of the Mathematics Framework for California Public Schools, p793.

[8] The mathematics of continuously compounded interest also has an analogue in the exponential representation of trigonometric functions.

[9] Generally, the standard deviation of such processes increases with the square root of time, while returns increase exponentially. Certain sequences of random variables are called martingales and are a central theme of advanced mathematical finance.

[10] See PISA 2018 Results https://www.oecd.org/pisa/Combined_Executive_Summaries_PISA_2018.pdf

[11] Goodman, J. (2019). The labor of division: Returns to compulsory high school math coursework. Journal of Labor Economics, 37(4), 1141-1182

[12] See, Greene, B. A., Miller, R. B., Crowson, H. M., Duke, B. L., & Akey, K. L. (2004). Predicting high school students' cognitive engagement and achievement: Contributions of classroom perceptions and motivation. Contemporary educational psychology, 29(4), 462-482.

[13] We discuss how mathematics engagement can be enhanced by financial applications in depth in our working paper "Enriching Mathematics through an Application to Finance" https://ficycle.org/wp-content/uploads/2021/12/Enriching-Math.pdf

[14] See momath.org/buck for a series of discussions of these connections.

[15] "They Just Add Up: Combined Math and Financial Knowledge Tied to Better Financial Outcomes" https://www.finrafoundation.org/sites/finrafoundation/files/Combined-Math-and-Financial-Knowledge-Tied-to-Better-Financial-Outcomes.pdf

[16] Chart taken from https://www.finrafoundation.org/sites/finrafoundation/files/Combined-Math-and-Financial-Knowledge-Tied-to-Better-Financial-Outcomes.pdf

[17] See, Urban, C., Schmeiser, M., Collins, J. M., & Brown, A. (2020). The effects of high school personal financial education policies on financial behavior. *Economics of Education Review*, *78*, 101786.