Financial Life Cycle Mathematics

Student Workbook Sample

UNIT 2: Earning Interest

Topic 1: Introduction to Earning Interest
Topic 2: Mathematics of Interest

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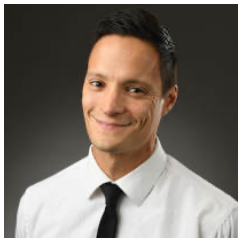
NOTE: This is a sample of two topics from our Unit 2 Workbook. This selection aims to offer a concise yet comprehensive overview of the diverse content covered in our course.

About the Authors

Jack Marley-Payne researches issues in education, finance and mathematics, and creates materials for teaching and understanding these topics. Jack grew up in the UK and moved to the US in order to obtain a PhD in philosophy from MIT. While there, they completed a doctoral dissertation focused on understanding knowledge in the light of psychological discoveries regarding our limited rationality, and taught undergraduate classes in philosophy and mathematics. Prior to this, they studied mathematics and philosophy at Exeter College, Oxford. Jack joined the FiCycle team in 2015, before it had been piloted in a single high school, and has been excited to see it become a thriving program, benefiting so many students.



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Andrew Davidson is a financial innovator and leader in the development of financial research and analytics, as well as the founder of FiCycle. He is president of Andrew Davidson & Co., Inc., a New York firm specializing in the application of analytical tools to investment management, which he founded in 1992. Andrew was instrumental in the creation of the Freddie Mac and Fannie Mae risk-sharing transactions, allowing them to attract private capital to bear credit risk, even as they remain in government conservatorship. Andrew is also active in other dimensions of GSE reform and has testified before the Senate Banking Committee on multiple occasions.

Acknowledgements: A great many wonderful people offered their time and wisdom to provide improvements to this book. Special thanks go to Andra Turner, Naiomi Israel, Alysia DaSilva, Bjorn Flesaker, Finn Kreidler, Gio Donovan, Jacob Katzman, Karie Parker Davidson, Karina Minchuk, Lauren Davidson, Paul Gray, Photo Anagnostopoulos, and Rose Davidson.

Table of Contents

How to use this Workbook

This workbook is the primary document you will use to learn about the material in unit 1 of FiCycle. It provides explanations of all the vocabulary and ideas you need to know, along with examples and exercises to help you cement your understanding.

You can read through the document page by page, doing the questions as you go, or use it as your teacher instructs.

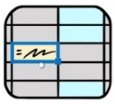
Here are the icons to look out for throughout the document:

Vocabulary: This introduces a new piece of vocabulary, either financial or mathematical, along with a definition of the term.

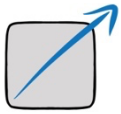
Question: This is a question you must answer. When you see this icon, it means you must do something. It's important you don't skip it!

Reflection Question: This is also a question that you need to answer. This time, though, it is an invitation to reflect, and does not have a single correct answer. You should feel free to write what comes to mind without worrying about getting it wrong.

Math Check: This is a worksheet to check you have the math skills you will need for the next section. Work through the questions to check and practice your skills. If you are not comfortable completing the sheet, ask your teacher for additional help.



Spreadsheet: This icon means that there is an activity you must complete on a separate spreadsheet document. There will be information on the document you need to access and instructions on what to do.



Extension: This is a question or activity you can do to deepen your knowledge of the material. It is an optional part of the course that you may choose to skip past if your teacher allows it.

Connection to the Life Cycle: This indicates a relationship between the topic you're working on and the overarching theme of the financial life cycle.

Financial Research Materials

In this course, you will have to research real financial products. This is practice for the real world where you don't just have to understand the financial products, you also have to know how to find the best deal.

Here is a list of some of the sites that will help you.

Financial Comparison Sites:

These sites allow you to compare various financial products meeting certain criteria – e.g. credit card with no annual fee. Comparison sites are a convenient way to shop around for the best deal without having to go through a large number of different websites by yourself. Some of the most common are:

https://www.nerdwallet.com/
https://www.creditkarma.com/
https://www.creditcards.com/

Be careful though, since these sites have sponsors, whose products they may promote. Make sure you check the details for yourself on any product you consider – don't just take the site's word for it when it says a product is your best option. More information on this can be found in the following article:

http://business.time.com/2012/08/10/do-credit-card-comparison-sites-work-as-promised/

Specialty Sites:

https://www.zillow.com/
This site provides information on homes, both to rent and to buy
You can search by area, price, size etc.
https://www.salaryexpert.com/salary/
This site provides information on the salary of various jobs
https://www.numbeo.com/
This site provides information on cost of living in locations around the world
It provides specific information on the cost of food, healthcare, and transit by location

As websites change over time, up to date links will be provided on the FiCycle website.

UNIT 2 INTRODUCTION

Looking Ahead

Over a financial life cycle, the income you receive at a given time will not always match what you need to consume, you can transfer money either forward or backward so you can consume it when you need it. You transfer money forward by investing and transfer it backwards by borrowing. Typically, you will be paid to transfer money forward and must pay to transfer it backward. Learning about investment strategy is a central part of the financial life cycle, and this requires understanding exponents and logs.

After completing this unit, you will be able to:

- Understand concepts related to earning interest
- Calculate simple interest
- Understand compounding
- Calculate compound interest
- Calculate continuously compounded interest
- Move between simple interest and continuously compounded interest
- Use the rule of 72
- Calculate present value using discounting
- Know the cost and benefits of the financial instruments used for borrowing and investing
- Understand the importance of credit scores
- Use interest calculations to create financial statements
- Apply exponents and logarithms to financial material

UNIT 2 TOPIC 1: INTRODUCTION TO EARNING INTEREST

You are just about to leave for school when a strange raggedy man shows up on your doorstep. He asks to use your bathroom and in return offers you a gift.

He is a time traveler and will give you something he took from the past. You have two options:

1. A set of solid gold jewel-encrusted eggs (known as 'Fabergé eggs') that he stole from Russian royalty during the 1917 revolution.
2. The rights to a bank account with Banca Monte dei Paschi di Siena, the world's oldest surviving bank, that he opened in 1472 when the bank was founded and that he hasn't touched since.

1. Which will you choose?

A set of Faberge Eggs – image source *http://www.saint-petersburg.com/famous-people/peter-carl-faberge/*

Some more information:

- The combined value of the eggs is over $300 million
- The time-traveler deposited a single dollar in the bank account when he opened it and has made no further deposits or withdrawals for 545 years.

2. Does this affect your choice?

Monte dei Paschi di Siena Headquarter's Main Entrance. Image source: *https://en.wikipedia.org/wiki/Banca_Monte_dei_Paschi_di_Siena*

Even more information:

Now you learn that due to a special introductory offer, this bank account was guaranteed 5% interest rate for as long as it remained open.

- Because of this, the value today after 545 years is over $350 *billion.*

Reminder on units:

- A **million** is a thousand times a thousand, so $1 million = $1,000,000
- A **billion** is a thousand times a million, so $1 billion = $1,000,000,000
- A **trillion** is a thousand times a billion, so $1 trillion = $1,000,000,000,000

If you want more practice dealing with these large units, look at our supplemental worksheet 'Unit 2 Topic 1 Math Sheet Millions, Billions, Trillions'

3. Did you make the right choice?

The reason the value of the bank account is so high is due to something called **compound interest**, which is the topic of this unit.

Transferring Money through Time: Borrowing and Investing

Over a financial life cycle, the income you receive at a given time will not always match what you need to consume; you can transfer money either forward or backward so you can consume it when you need it.

Investing is when you transfer money forward *to* your future self.

Borrowing is when you transfer money backwards *from* your future self.

- Typically, you will be paid to transfer money forward, and must pay to transfer it backward.

An **investment** is money transferred to the future from the present by means of a financial product**.**

- When people have money that is not needed for immediate expenses, they may choose to invest it (this extra money is known as a **surplus**).
- They transfer their **surplus** to someone who needs cash to meet immediate needs – either a bank or another financial company, a government, or an individual.
- When investing, one expects to receive one's money back at a later date and to be paid for the service in the form of receiving more money than initially invested.
- Common reasons to invest include saving money for retirement, for a down payment on a house, or for a child's education.
- Typically, the purpose of investing is to help one meet future financial goals.

A **loan** is money transferred to the present from the future by means of a financial product.

- A **loan** is a financial instrument that lets you *borrow* money from your future self.
    - In other words: A loan is a tool that allows you to spend your future income in the present.
- Sometimes you do not have money available to meet a current financial need – either to purchase an asset or cover expenses.
- Through a loan you may acquire money from a company, a bank, or an individual to meet such needs.
- You will have to return the money as well as pay the lender for the service.
- Common reasons to borrow include paying for education, paying for a house, and paying for a car.

Because of the workings of loans and investments, the value of money invested or borrowed changes over time.

- Money increases in value if saved for the future. For example, you loan your money to a person or institution and they pay you for the use of your money.

Interest is the additional value you pay or receive over time when you borrow or invest.

We use specific terminology for interest when borrowing and investing:

- When you invest, you **earn** interest – earning interest is a form of income.
- When you borrow, your loan **accrues** interest – accruing interest is a form of expense.

The initial amount of money deposited or invested is known as the **principal**.

[Word Bank: more; less; loan; investment; future; accrue; pay; surplus; past; deficiency; loan; investing; borrowing]

4. If you now possess money that you don't need it's called a _____. You can transfer this money to the future by _____.
5. You borrow money if you need to spend _____ income now.
6. In order to borrow money, you must take out a _____.
7. When you invest, you generally receive _____ money than you put in, but you receive it at a later date.
8. If you borrow money, you will _____ interest.
9. If you invest, you will _____ interest.
10. Decide whether each of the following financial goals will likely be met by investing, borrowing, or a bit of both. Explain your answer.
    a. Having money available when you retire

    b. Starting up a new business

    c. Paying college fees

    d. Providing an inheritance for your children.

11. Provide your own example of a financial goal that should be met through investing – explain your answer.

12. Provide your own example of a financial goal that should be met through borrowing – explain your answer.

Value and Time

This unit will look at the effects of interest and its role in the financial life cycle. To do this, we need three key concepts: (1) future value; (2) present value; (3) interest rate.

Future value (FV) is the value a given sum of money will be worth at a certain point in the future.

Example: You invest $1000 for 3 years – the value of your investment after 3 years is its **future value.**

Present value (PV) is the *current value* of a given sum of money that will be received at some point in the future.

Example: Your friend agrees to pay you $80 in six months' time, if you pay her $50 now – the **present value** of the loan is $50.

13. You invest $200 in a savings account and after 5 years the account balance is $230.
    a. What is the present value of your investment?

    b. What is the future value of your investment?

14. You invest $35 in your cousin's startup, and after 5 years the startup has tripled in value.
    a. What is the present value of your investment?

    b. What is the future value of your investment?

Interest rate (r) is the percentage of money you earn or pay for investing or borrowing each year.

- The interest you earn or pay is proportional to the amount of money you invest: you earn interest as a *percentage* of the original amount. This percentage value is called the **interest rate**.
- *Notation*: We will express an interest rate where you earn five percent of the amount invested as "5%".

Example: You invest $100 with an interest rate of 3%. This means that the interest you earn is 3% of $100: $3.

15. You borrow $75 with a 60% interest rate – what is the interest on your loan?

A note on time: The interest rate is an **annual** measure – you receive the interest after a year of investing (or borrowing.)

Example: Ava invests $1000 in a savings account for a year. The interest rate is 1% Let's look at what happens after a year.

- After a year, Ava will be paid her interest, which is the interest rate multiplied by the present value: $1000 \cdot 0.01 = 10$
- So, after a year, Ava will have $1010.
- This means that the future value of the investment is $1010 – we write FV = 1010.
- Conversely, the present value of the investment is $1000 – we write PV = 1000.

16. Carlos borrows $250. The interest rate is 30%. How much will Carlos owe after a year has passed?

Next Question: What happens if you invest or borrow over more than one year?

- To answer this, we need to look at the *mathematics of interest* in the next section.

Unit 2 Topic 1 Check for Understanding

Section 1. Complete the following definitions, using the word bank below:

[Word Bank: accrue, backward, earn, future-value, lend, forward, borrow, interest, principal, present-value, loan, less, more]

1. Interest is earned when you transfer consumption _____ in time.
2. Investments _____ interest, while loans _____ interest.
3. If you want to know how much your investment will be worth after a certain time, you are interested in its _____ .
4. An investment's _____ is how much it's worth now.
5. Investments are transactions in which you _____ excess income to other institutions.
6. The price you pay to use borrowed money is called _____.
7. The fundamental principle of the time value of money is that dollars received today are worth _____ than dollars promised in the future.
8. If you want to use future income for present expenses, you need a _____.

Section 2. Circle all correct answer(s). There might be more than one correct answer:

9. People transfer money forward in time because:
   a. The government makes them
   b. They want to consume more than they will earn in the future
   c. They don't currently want to buy anything
   d. They like to describe themselves as an investor to their crush

10. If you invest with an annual 5% interest rate, *when* will you get the 5% interest?
    a. After you ask for it enough times
    b. When you truly need it
    c. After you do 1000 push-ups
    d. After a year

11. Which of the following are good reasons to borrow money?
    a. You think we're near the end of days, so you will never have to pay it back
    b. So you can buy an asset that will reduce expenses in the long run
    c. To pay for courses to earn a college degree that will increase your income
    d. Your friends will be impressed if you have a lot of cash in your wallet

Section 3. Show all work.

12. Elise borrows $1000. The interest rate is 15%.
    a. How much interest will accrue in one year?

    b. How much money will Elise owe in one year?

13. Ronald borrows $1,000,000 from his father. Since they are family, his father gives him an amazingly low interest rate of 0.3%.
    a. How much interest will Ronald accrue in one year?

    b. How much money will Ronald owe in one year?

14. Harry borrows $1,234 from Dudley with an interest rate of 5.6%.
    a. How much interest will Harry accrue in one year?

    b. How much money will Harry owe in one year?

UNIT 2 TOPIC 2: MATHEMATICS OF INTEREST

Math Check 1: Exponents

To understand the material in this topic, you must be comfortable with exponents – test your knowledge with the following questions:

- *Note*: You also need to be comfortable with fractions to answer these questions. If you need help with this, ask your teacher.

Section 1. Multiplying Powers

Example: $x^4 \cdot x^3 = x^7$

General Rule: $a^m \cdot a^n = a^{m+n}$

1. $z^2 * z^7 =$
2. $3xy^3 \cdot 6x^4y^8 =$

Section 2. Dividing Powers

Example: $\frac{x^7}{x^2} = x^5$

General Rule: $\frac{a^m}{a^n} = a^{m-n}$

3. $3.\frac{y^{16}}{y^9} =$
4. $\frac{q^3r^8}{qr^2} =$

Section 3. Raising a Power to a Power

Example: $(x^8)^3 = x^{24}$

General Rule: $(a^m)^n = a^{m*n}$

5. $(x^4)^{12} =$
6. $(x^6)^4 5y^7 =$

Section 4. Distribution of powers

Example: $(xy)^4 = x^4y^4$
General Rule: $(ab)^n = a^n b^n$

7. $(xz)^{19} =$

8. $(5x^6y^3)^2 =$

Section 5. Zero Exponents

General Rule: $a^0 = 1$

9. $\frac{y^{28}}{y^{28}} =$

10. $7x^2z^0 \cdot 3x =$

Simple Interest

It is important that we can calculate the future value of an investment, given information about principal and interest rate.

- There are two different ways of earning interest: (1) **simple interest** and (2) **compound interest**.

We'll start by looking at **simple interest**.

Simple interest over a year: Over one year, with interest rate r, you earn r of your deposit.

- For example, if r = 5%, you would earn 5% of the amount you deposited. Thus, any amount you deposit now will be worth an additional 5% in the future.

Example: You invest $1000 with interest rate of 8%, with **simple interest**. How much will you have after a year?

- After a year you will receive $1000 \cdot r = 1000 \cdot 0.08 = 80$ in interest.
- Since you still have your initial $1000, you will have $1080 in total.

11. Identify the values of the following variables in the example above.
    a. Future Value: FV =_____
    b. Present value: PV =_____
    c. Interest rate: r =_____

In general, after one year, the value of your investment is the sum of the initial value (PV) and that initial value multiplied by the interest rate (r). To put it symbolically:

- $FV = PV + r \bullet PV = PV \bullet (1 + r)$ [We factor out PV to simplify]

Simple interest over multiple years: If you earn interest on your deposit over multiple years, you will earn r of your deposit multiple times.

Example: You earn 8% interest on $1000 each year for 3 years; you end up with your initial $1000, along with three sets of 8% of $1000

- That is, you'll have $1000 + 1000 \bullet 0.08 \bullet 3 = 1000(1 + 0.08 \bullet 3)$; which is $1240.

12. You invest $245 with interest rate 14.6%. How much will you have after earning simple interest for 2 years?

We can see a general pattern here: if you invest with simple interest for a certain number of years, you end up with that number of sets of the rate multiplied by the principal (along with the principal itself).

- Symbolically, if you invest for t years, you will end up with $PV + PV \cdot r \cdot t$; this gives us the **simple interest formula**:

Simple Interest Formula: $FV = PV \bullet (1 + rt)$

Formula Key:
FV = Future Value
PV = Present Value
r = Rate
t = Number of years

13. You invest $950 for 12 years with a simple interest rate of 5.3%. How much will you end up with? Use the simple interest formula (above), and state what values you assigned to each variable.

14. You invest $11 for 48 years with a simple interest rate of 9%. How much will you end up with? Use the simple interest formula, and state what values you assigned to each variable.

15. You invest $30,000 for 6 years with a simple interest rate of 15%. How much will you end up with? Use the simple interest formula, and state what values you assigned to each variable.

Compound Interest

Usually if you invest over multiple years, you will earn *interest on your interest*, this is known as **compounding**.

- Each year you earn r of the amount you have at the end of the previous year (including both your principal *and* your interest from the past year)

Example: You invest \$50 for two years with compound interest at a rate of 10%.

- In the first year, you earn 10% of **\$50**
  - 10% = 0.1
  - 0.1 • 50 = 5
  - 50 + 5 = 55 → Therefor you have \$55 at the end of the first year
- In the second year, you earn, 10% of **\$55**
  - 10% = 0.1
  - 0.1 • 55 = 5.5
  - 55.0 + 5.5 = 60.5 → Therefor you have *\$60.50 at the end of the second year*

16. How much will you earn if you invest \$125,000 for two years, with a compound interest rate of 4%?

Now let's look at things symbolically, using *PV* for present value, *FV* for future value, *r* for interest rate, and *t* for time. In the table below we will start with the present value of a loan and calculate interest year over year to see how compound interest works.

Formula	**Explanation**
PV	The Present Value of your investment.
$PV + PV \bullet r$ $PV \bullet (1 + r)$	This is the value of your investment after one year. We simplify by factoring out *PV*.
$PV \bullet (1 + r) \bullet r$	This is the interest you earn after the second year – we take the total at the end of one year and multiply by *r*.
$PV \bullet (1 + r) + PV \bullet (1 + r) \bullet r$ $PV \bullet (1 + r) \bullet (1 + r)$ $PV \bullet (1 + r)^2$	This is the total value of your investment after two years. We simplify by factoring out *PV* and (1+*r*). We simplify using the laws of exponents.

$PV \bullet (1+r)^n$	Suppose after n years this is the value of the investment. We want to know its value after another year (n+1 years).
$PV \bullet (1+r)^n + PV \bullet (1+r)^n \cdot (1+r)$ $PV \bullet (1+r)^n(1+r)$ $PV \bullet (1+r)^{n+1}$	We sum the initial amount with that amount multiplied by r. We factor out $PV \bullet (1+r)^n$. We simplify using the laws of exponents.

This tells us that in general, when you earn compound interest for t years, the following equation holds:

Simple Compounding Interest Formula: $FV = PV \bullet (1+r)^t$

Formula Key:
FV = Future Value
PV = Present Value
r = Rate
t = Number of years

Note: This is a special instance of the **compound interest formula** that we will discuss in topic 2.

Example: Suppose you invest $1000 for 3 years at a rate of 8%, then:

$$FV = PV \bullet (1 + r)^t$$
$$FV = PV \bullet (1 + .08)^3$$
$$FV = 1000 \bullet (1.08)^3$$
$$FV = 1259.71$$

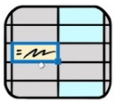
We calculated above that if you earned **simple interest** of $1000 at 8% for three years, you would have had $1240, so earning interest on your interest got you an additional $19.71!

17. You invest $450 for 30 years with compound interest at rate of 9.7%. What is the future value of your investment?

18. You take out a bad loan for $900 with an interest rate of 28%. How much do you owe after 15 years i you don't make any payments?

19. Rose has access to an account that earns 5% interest every year (it compounds annually). She reasons that if she deposits $100, she will make $10 after two years and have a total of $110. Is she correct? Explain.

Note: In almost all real-world investments, interest over multiple years is compounded, not simple. **Unless otherwise stated, assume interest is compounded in all examples and problems in this book.**



Spreadsheet Connection

You need to be able to calculate future value on Excel – to learn how to do this, go through the work sheet 'Unit 2 Exponents and Simple Compounding in Excel'.

20. What is your entry in cell B6 of sheet 2?

21. What is your entry in cell E5 of sheet 4?

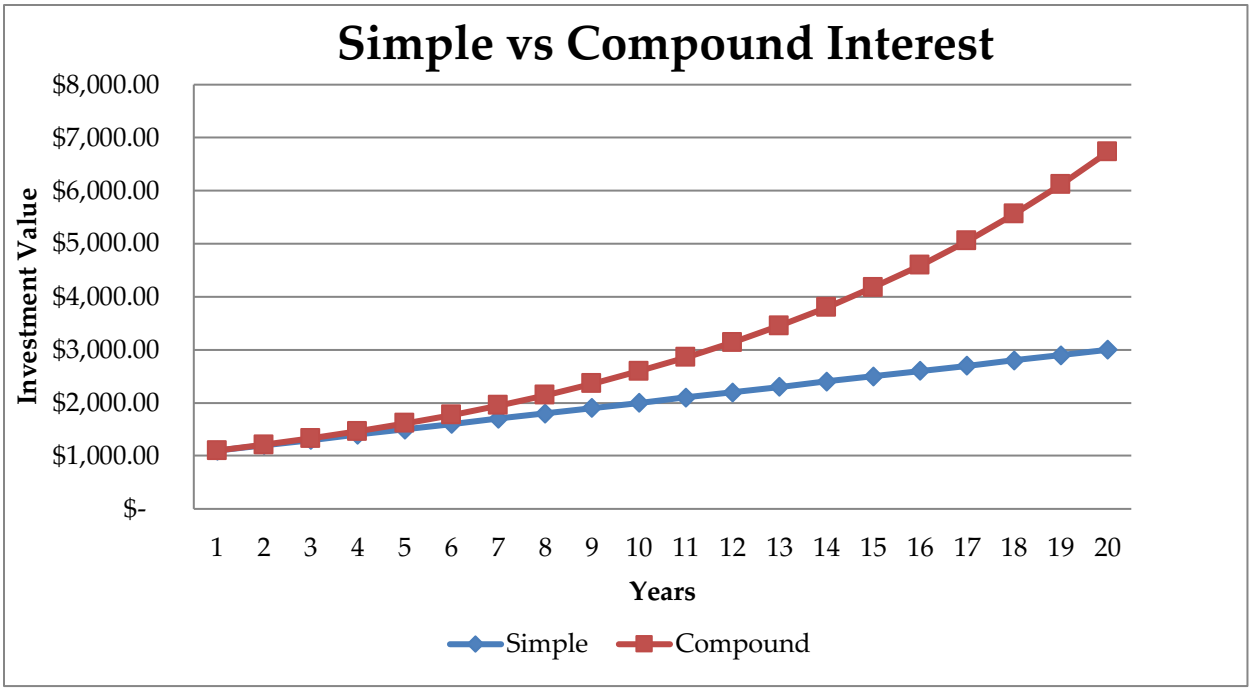
Comparing Simple and Compound Interest

You are investing $1000 with 8% interest; compare what happens if the interest is simple vs compound, using the chart below.

We can see that the value of your investment is greater over time when there is compound interest.

- This is not a big surprise since with compound interest you get interest on your interest, on top of what you already get with simple interest.
- Note how dramatic the increase is: after 20 years, your investment is worth over $2,000 more with compound interest than with simple interest.

Year	Simple Interest	Compound Interest
1	$ 1,080.00	$ 1,080.00
2	$ 1,160.00	$ 1,166.40
3	$ 1,240.00	$ 1,259.71
4	$ 1,320.00	$ 1,360.49
5	$ 1,400.00	$ 1,469.33
6	$ 1,480.00	$ 1,586.87
7	$ 1,560.00	$ 1,713.82
8	$ 1,640.00	$ 1,850.93
9	$ 1,720.00	$ 1,999.00
10	$ 1,800.00	$ 2,158.92
11	$ 1,880.00	$ 2,331.64
12	$ 1,960.00	$ 2,518.17
13	$ 2,040.00	$ 2,719.62
14	$ 2,120.00	$ 2,937.19
15	$ 2,200.00	$ 3,172.17
16	$ 2,280.00	$ 3,425.94
17	$ 2,360.00	$ 3,700.02
18	$ 2,440.00	$ 3,996.02
19	$ 2,520.00	$ 4,315.70
20	$ 2,600.00	$ 4,660.96



22. How would you describe the pattern of the two lines on the graph?

23. Does the difference in value between an investment with compound and simple interest increase, decrease, or stay the same over time?

24. Does the rate of change for an investment with simple interest increase, decrease, or stay the same over time?

25. Does the rate of change for an investment with compound interest increase, decrease, or stay the same over time?

The crucial difference is that with simple interest, each year you add $r \cdot PV$ to the previous total, while with compound interest, you *multiply* the previous total by $(1+r)$.

- Since we are repeatedly **adding** when dealing with **simple interest,** there is **linear growth.**
- Since we are repeatedly **multiplying** when dealing with **compound interest,** there is **exponential growth**.

Linear growth occurs if a value increases by adding a constant amount each period. **Exponential growth** occurs if a value increases by multiplying by a constant ratio each period.

You are offered an investment with simple interest at a rate of 10% or compound interest at a rate of 8.5%. You have $9,000 to invest. Based on this, answer numbers 26 to 28 below.

26. Fill in the table for the value of your investment over 10 years, for each option.

Year	Simple Interest	Compound Interest
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

27. Sketch a graph showing the values of the two potential investments over time in the table above; it does not have to be precise.

28. Using the graph you created, for which periods of investment (if any) would the simple interest option be better? For which periods of investment (if any) would the compound interest option be better?

Note: No matter how much greater the interest rate for simple interest is than the rate for compound interest, eventually the investment with compound interest will outgrow the one with simple interest.

- This is because an exponential function will always outgrow a linear function eventually.

Unit 2 Topic 2 Check for Understanding

Section 1. Complete the following definitions:

[Word Bank: longer, exponential, greater, compound, smaller, simple, linear, interest]

1. _____ interest is when you only earn interest on the initial amount you invested.
2. When compounding over multiple years you can earn interest on your _____.
3. An investment with simple interest has _____ growth while an investment with compound interest has _____ growth.
4. With compound interest, the longer you invest for, the _____ the interest earned at the end of the final year.

Section 2. Simple Interest. Show all work.

5. Calculate the interest that will accrue when $600 is borrowed for one year with a simple interest rate of 8%.

6. Karen borrowed $2,500 from her brother to help her put a down payment on an apartment. She agreed to repay her brother in two years with 5% simple interest. How much will Karen owe her brother in two years?

7. Sarah deposits $1300 in a savings account with an interest rate of 1.2%. How much does she have after 4 years with simple interest?

8. Milo has $4200 to invest and aims to have $5500 in 5 years time. What simple interest rate would he need to achieve this?

9. Paolo has $6250 after investing for 3 years with a simple interest rate of 3.6%. How much did he initially invest?

Section 3. Compound Interest. Show all work.

10. Kelly invests $25,000 for 15 years with compound interest at a rate of 6.26%. What is the future value of her investment?

11. Geoff deposits $1750 in a savings account with an interest rate of 2.9%. How much does he have after 4 years with compounding?

12. Dan borrows $800 with an interest rate of 11%. How much does he owe after 3 years with compounding?

13. Sara invests $5000 with an interest rate of 4.5%. She also takes out a loan for $1900 with an interest rate of 10.4%. Both are subject to compound interest for five years.
    a. What is the future value of the investment?

b. What is the future value of the loan?

c. What is Sara's net income over five years (ignoring all other transactions)?

d. Suppose Sara had not taken out the loan and instead had invested $1900 less. What would her net income have been then? In light of this, what advice would you offer Sara?

Section 4

You are offered an investment with simple interest at a rate of 15%, or compound interest at a rate of 13%. You have $9000 to invest.

14. Fill in the table for the value of your investment over 10 years, for each option.

Year	Simple Interest	Compound Interest
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

15. Sketch a graph showing the values of the two potential investments over time.

16. For what periods of investment (if any) would the simple interest option be better? For what periods of investment (if any) would the compound interest option be better?

Extension

17. Jennifer wants to invest $3000 and aims to have at least $3700 in 3 years' time. With compounding, what interest rate would she need to achieve this?

Spreadsheet Connection

Now complete 'Unit 2 Topic 2 Check Sheet'.

NOTE: This is a sample of two topics from our Unit 2 Workbook. This selection aims to offer a concise yet comprehensive overview of the diverse content covered in our course.

Proceed to view Unit 2 Summary and Glossary

UNIT SUMMARY

This unit explains the effects of transferring consumption over time.

- You must pay to accelerate consumption, and you will be paid to transfer consumption to the future.
- Value increases exponentially over time, due to **compounding.**

When looking to invest or borrow, there are three key factors you need to consider.

- Interest rate
- Ease of access (or flexibility)
- Risk

To get more favorable loans, you must have a good **credit score.**

Here are the most important formulae from Unit 2:

Compound Interest Formula:

$$FV = PV \bullet (1 + \frac{r}{n})^{n \cdot t}$$

Continuous Compounding Formula:

$$FV = PV \cdot e^{rt}$$

Discounting Formula:

$$PV = FV \bullet (1 + \frac{r}{n})^{-n \cdot t}$$

Simple Compounding to Continuous Compounding:

$$PV \bullet e^{rt} = PV \bullet (1 + R)^t, r = \ln(1 + R)$$

Rule of 72:

$$t = \frac{72}{R \cdot 100}$$

Formula Key:

FV = Future Value
PV = Present Value
r = Interest rate
n = Reinvestments per period
t = Periods of investment
R = Equivalent interest rate with simple compounding

Mathematics Content Review: To master the financial content in this unit, you must understand the concepts and procedures associated with, and be able to complete the worksheets related to:

- Logarithms Worksheet (found in Topic 4)
- Exponents Rules Worksheet (found in Topic 2

GLOSSARY

APR: Stands for 'Annual percentage rate' – it is a government regulated measure of annual interest rate. It takes the interest payments over a year, including any fees and compounding, and gives the equivalent as a *simple* annual interest rate.

Bank Loan: When a bank lends you an agreed upon sum of money, usually with an agreed upon interest rate, and provides an agreed upon schedule for paying it back

Bonds: Financial product issued in return for a loan to a government or institution. Owning a bond entitles you to periodic payments, as well as full repayment when the loan ends (this is called maturity).

Borrowing: When you transfer money backwards from your future self

Cash Back: A common type of credit card reward in which you get a percentage of your spending amount refunded on your card

CD Account: Investment instrument provided by a bank. You loan the bank a sum of money for a fixed period (for example 5 years) and there is a fee for withdrawing the money early. In return, there is typically a higher interest rate than for a savings account.

Compounding: When you earn interest on the interest earned in previous periods.

Compounding formula: $FV = PV \bullet (1 + r)^t$

Compound interest: When interest is compounded, you take your interest, and reinvest multiple times during the period in order to earn interest on your interest. The formula for compound interest is $\text{FV} = \text{PV} \bullet (1 + \frac{r}{n})^n$, where n is the number of times you reinvest during the period.

Continuous compounding: This is when one takes the limit of what can be earned through compounding. That is $\text{FV} = \lim_{n \to \infty} PV \left(1 + \frac{r}{n}\right)^n$. Equivalently $FV = e^{rt}$

Continuous compounding formula: $FV = e^{rt}$

Credit Score: A tool used by lenders to assess the risk that a customer will be unable to repay them on time

Discounting: The process of calculating the present value of a given sum paid in the future

Discount Rate: The rate used to calculate present value – an alternative term for **interest rate.**

Exponential Growth: When a value increases by multiplying by a constant ratio each period

Financial Product: A tool created by a company that allows you to meet certain financial goals, typically investing and borrowing

Future value (FV): The value a given sum of money will be worth at a certain point in the future.

Grace Period: Time period within which you are not charged interest on credit card balance (typically one month).

Interest: The additional value that money invested or borrowed acquires over time.

Interest rate (r): Percentage rate that determines the value of interest added after each period.

Investing: When you transfer money forward to your future self

Investments: Money transferred to the future by means of a financial product.

Linear Growth: When a value increases by adding a constant amount each period

Loan: Money transferred to the present from the future by means of a financial product

Money market account (MMA): Account that pays interest based on current rates in money markets.

Present value (PV): The value now of a sum of money that will be received or paid at some point in the future.

Principal: The initial amount of money deposited or invested.

Promotional Rates: More favorable interest rates that apply for a limited time when you first get a credit card.

Return: The amount received from an investment.

Savings Account: Investment tool offered by banks. You loan a sum of money to a bank for an unspecified period): they have a low interest rate, are low risk, and give you easy (almost) immediate access to money.

Simple interest: When interest payments are made only on the principal, not on previously earned interest.

Simple interest formula: $FV = PV \bullet (1 + rt)$

Student Loan: A bank or the government lends you an agreed upon sum of money for the explicit purpose of covering the costs of college

Time-value of money: The changing value of a sum of money at different points in time. The value of an investment is generally expected to grow over time.