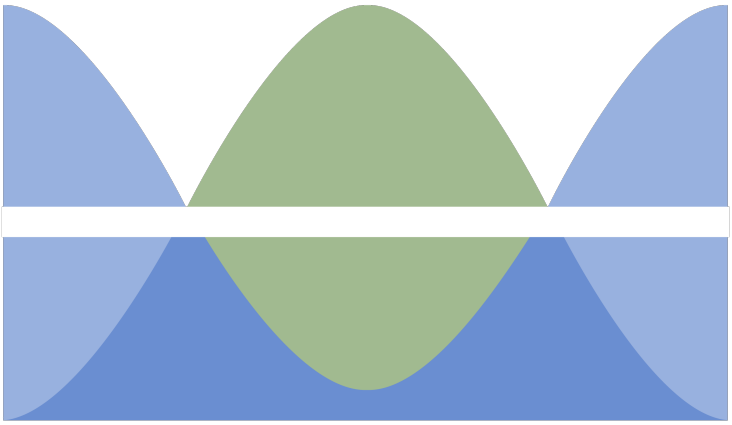
Financial Life Cycle Mathematics



FICYCLE

Teacher's Packet

Welcome to FiCycle! FiCycle combines finance and mathematics into an interesting course that empowers students to take control of their financial lives through the use of mathematics. This document provides an overview of the FiCycle course and materials to help make sure you have all the tools you need to teach FiCycle to your students.

Document contents:

1. Introduction to the course
2. Guide to the course materials
3. Getting organized to teach the course
4. Snap shot scope and sequence
5. Common core and Jumpstart standards overview
6. Unit Plan for each unit

Introduction

Financial Life Cycle Mathematics (FiCycle) is a high school course in financial math providing students the fundamental concepts in personal finance, using the necessary math to provide a versatile theoretical understanding.

The math content is roughly at the level of Algebra II, but can easily be modified to that of Algebra I. Students taking the course are typically high school juniors and seniors who are becoming more and more focused on their future.

Course Structure

The course contains five units and three alternative units:

- **Unit 1: Financial Statements** – Students learn about wealth by creating income statements, balance sheets and budgets. This incorporates the mathematics of spreadsheets and a review of Algebra I.
- **Unit 2: Earning Interest** – Students learn how transferring money to the future increases value through compounding. This incorporates the mathematics of exponents, exponentials and logarithms.
- **Unit 3: Regular Payments** – Students learn the math underlying regular cash flows such as mortgages and retirement savings. This incorporates the mathematics of sequences, series and limits.
- **Unit 4: Insurance and Expected Value** – Students are introduced to risk and making decisions in the face of uncertainty. This incorporates the mathematics of probability and expected value.
- **Unit 5: Stocks and Risk** – Students learn about the stock market, with a focus on the efficient market hypothesis and the statistics related to diversified and systemic risk. This incorporates the mathematics of correlation and normal distributions.
- **Alternative Unit: The Role of Government** – Students gain an understanding of the government's role in shaping the environment in which individuals make financial decisions. This involves applying mathematics to real-world situations.
- **Alternative Unit: Trigonometry and Complex Numbers** – Students learn complex numbers and trigonometric functions of the unit circle. This builds on the mathematics of compound interest to develop functions of complex numbers on the unit circle.
- **Alternative Unit: Advanced Extensions** - Students investigate more advanced mathematical models of the financial topics discussed in the previous five units. This may provide advanced students with a strong foundation to go on to a first course in calculus.

Course through-lines for students

1. What is the role of the financial life-cycle in influencing our financial decisions?
    a. How do financial instruments help transfer consumption across time?
    b. How can we measure and manage financial risk?
2. What is the role of math in guiding financial decisions?
    a. How can algebra help us model, understand, and predict the effects of transferring consumption?
    b. How do probability and statistics improve our understanding of managing risk?

Materials Guide

We provide FiCycle teachers with access to a Dropbox folder which contains a complete set of course materials. The course materials are divided into units. For each unit there is a folder that contains the following materials:

1. **Outline:** Provides a scope and sequence, essential questions and common core state standards for the unit
    - Location: ***Unit Folder***
2. **Student Workbook:** Explains the material for each unit, with examples and problems suitable for students
    - Location: ***Unit Folder***
3. **Topic Quiz:** Questions assess essential knowledge for each topic
    - Location: ***Quizzes Folder*** within the *Unit Folder*
4. **Math Sheet:** Worksheets that remediate and asses the math component of each unit
    - Location: ***Math Sheet Folder*** within the *Unit Folder*
5. **Spreadsheet Work Sheets:** Explains how to use the relevant spreadsheet tools for each unit
    - Location: ***Spreadsheet Materials Folder***, within the *Unit Folder*
6. **Additional Instructional Materials:** Additional practice questions for difficult topics, games and activities for the classroom
    - Location: ***Instructional Materials Folder***, within the *Unit Folder*
7. **Project:** An end of unit take home project that requires analysis of a realistic financial scenario
    - Location**: *Project Folder,*** within the *Unit Folder*

Material format: Materials are provided as Microsoft Word and Excel documents that teachers are encouraged to re-format and adapt. The Excel documents are Google Sheets compatible.

<u>Supplementary Materials</u>:

The following materials may assist in teaching the course if needed.

Personal Finance – Rachel Seigel

- This provides a comprehensive guide to the financial terminology that might come up in the course – it will be useful for teachers to consult regarding the course's financial details.

Mathematical Finance – M.J. Alhabeeb

- This book goes through more mathematically sophisticated material that extends the topics discussed in the course – it will be a useful source of questions for more advanced students.

Smart Choices – Hammond, Keeney and Raiffa

- This provides a systemic look at good decision making, which will help students understand how to approach financial decisions.

Teaching the Course

Organizing for the Course

FiCycle was designed to meet the needs of many different types of schools and students. The curriculum is easily adaptable and can be incorporated into the high school curriculum in a variety of ways.

FiCycle can be taught as a year-long course for Juniors and Seniors who have completed Algebra and some Advanced Algebra and who need an additional math course.

- FiCycle will reinforce Algebra math concepts and extend students' knowledge of probability, modeling and mathematical thinking.

FiCycle can be taught as a year-long course for students who have only completed their basic Algebra requirements.

- It will apply familiar elements of Algebra I in a new context, and students will learn Algebra II math as they go

FiCycle can be offered as a half year course combined with trigonometry or other math class for the other half year.

- Units 1-3 form a self-contained set of topics
- Some schools may want to use Units 1, 2 and 4 for a semester course

Advanced students taking or prepared for pre-calculus and calculus maybe able to complete units 1 to 5 during a semester course.

- Units 1-3 form a self-contained set of topics
- The FiCycle workbook can be used for self-paced learning

We provide a range of materials to support teachers, but we believe teachers can exercise their discretion in determining **how** the course is taught.

- Material in the workbook can be used as the basis for group activities, class discussions and presentations, or individual study, depending on preference
- When students can successfully complete the quizzes and projects, they will have the requisite understanding.

Assessment

Each unit has a final project as its primary assessment. This project presents a description of a character facing a particular financial problem. The student must analyze this situation and advise the character on what they should do, while making the necessary mathematical calculations to back up their advice. There are few correct or incorrect answers, rather students use mathematics and their knowledge of various financial instruments to make informed decisions and give advice using mathematics as their evidence.

- The topics covered in the unit provide students with the knowledge and skills to complete this project.
- The workbook provides explanations, examples, and exercises for these topics
- The quizzes provide weekly summative assessments of the material

Teacher Support

We hold training workshops for all first time FiCycle teachers in the summer to introduce them to the materials and make sure they are comfortable with Units 1-3.

We also offer training before the start of spring semester to help teachers with Units 4-6.

We are available via email at info@ficycle.org to answer any questions from teachers throughout the year.

Alignment with Common Core Standards

The modeling standards and the mathematical practice standards are covered across the whole course.

Unit	Algebra	Functions	Statistics & Probability
1	A-CED.1; A-CED.2 A-CED.4; A-REI.1 A-SSE.1	F-IF.1 F-IF.2	
2	A-CED.1; A-CED.2 A-CED.4; A-REI.1 A-REI.10; A-SSE.1		
3	A-APR.1; A-CED.2 A-REI.1; A-REI.10 A-SSE.1; A-SSE.2 A-SSE.4	F-BF.1 F-BF.2 F-LE.2 F-LE.5	
4			S-CP.1-9 S-MD.1-7
5			S-IC.1-9 S-MD.1-7
6			

Alignment with Jump Start Standards

Unit	Spending and Saving	Credit and Debt	Employment and Income	Investing	Risk Management and Insurance	Financial Decision Making
1	1, 2, 3, 4	3	1, 2, 3			1, 2, 4, 5, 8
2	1, 4	1, 2, 3		1, 2, 3		2, 4, 5, 8
3	1, 4	3	2, 3			1, 2, 4, 5, 8
4					1, 2, 3	2, 4, 5, 8
5	1, 4			1, 2, 3		2, 4, 5, 8
6						

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Unit 1 Financial Statements	Unit 2 Earning Interest	Unit 3 Regular Payments	Unit 4 Insurance and Expected Value	Unit 5 Risk and Stocks	Supplementary Units
Topic 1 – Understanding wealth • Introduction to concepts: income; expense; asset; liability • Appreciation and depreciation Topic 2 – T Accounts • Cash, Other Assets, Liabilities, Income, Expense • Balancing transactions Topic 3 – Income Statements • Introduce income equation; recap linear equations • Introduction to excel: creating tables, basic arithmetic functions • Connecting income statements and T-Accounts Topic 4 – Balance Sheets • Accounting equation and balance sheets • T-Accounts and beginning and ending balance sheets Topic 5 – Sources and Uses • Introduction to liquidity • Creating sources and uses statements • Relate to T-Accounts Topic 6 – Budgeting • Introduce budgeting • Create budgeting strategies Topic 7 – Tax • Introduce tax – show place on financial statements • Calculate tax rates given income • Fill out 1040	Topic 1 – Introduction to Earning Interest. • Discuss transferring consumption across time: Investing and borrowing Topic 2 – Mathematics of interest • Introduce compounding • Recap the rules of exponents • Linear and exponential growth Topic 3 – Compound Interest • Explain the principle of compounding within a period • Compound interest formula: FV=PV(1+r/n)^nt • Introduce e (Euler's number) • Continuously compounded interest: FV=PVe^rt • Limits Topic 4 – Rule of 72: Double your money • Introduce logs • Convert continuously compounded to simple interest • Introduce rule of 72 Topic 5 – Present Value • Explain Accelerating Consumption • Discounting equation: PV=FV*(1+r/n)^-nt Topic 6 – Earning interest in the financial life cycle • Different financial instruments transfer consumption with different costs/benefits Topic 7 – Credit Scores • Explain role of credit score • Discuss measures to improve your credit score	Topic 1 – Arithmetic Series • Learn to work with arithmetic series & sequences Topic 2 – Geometric Series • Learn to work with geometric series & sequences • Calculate future and present value of annuity • Introduce concept of saving/paying off loans in installments Topic 3 – Annuities • Calculate future value of annuity • Calculate present value of annuity Topic 4 – Mortgages • How to calculate mortgage value • Advantages/disadvantages of a mortgage • Creating amortization schedule Topic 5 –Growing cash flow • Examples of growing payments. • Dividend discount model (d/r-g) • Apply geometric series formula to calculation of growing payments Topic 6 – Retirement • Concept of retirement in relation to the financial life cycle • Calculating how much is needed for retirement Topic 7 – Interest Rates and Inflation • Show how Inflation acts like a negative growth rate • Calculate inflation adjusted future value	Topic 1 – Introduction to insurance • Risk in the financial life-cycle • Insurance as a tool for avoiding risk (health; vehicle; property; life) • Types of insurance (deductibles and copays) Topic 2 – Probability • Compound experiments • Independent events • Tree diagrams Topic 3- Expected Value • Expected value and decision making Topic 4 – Expected Value and Insurance • Calculating EV of insurance • Expected value vs. expected utility • Comparing EV and EU for insurance Topic 5 – Diversifying risk • Definitions of diversified and stubborn risk • Dice game • Applications to insurance Topic 6 – The Binomial theorem • Binomial trees • Choice function • Binomial theorem • Applications to insurance	Topic 1 – Introduction to Stocks • Definition of stocks • Income from stocks: capital gain; dividend • Expected value of stocks Topic 2 – The efficient market hypothesis • New information affects share price • Over-valued and under-valued stocks • The efficient market hypothesis Topic 3 – Random Walks Topic 4 – Risk and portfolios • Defining portfolios and portfolio return • Portfolio expected value and risk Topic 5 – Normal distribution and standard deviation • Introduce standard deviation • Calculate population and sample standard deviation on calculator/excel • Normal distribution Topic 6– Normal distribution and Financial Risk • Binomial distributions approximate normal distribution • Mean and SD for binomial distributions • Applications for financial portfolios Topic 7 – Risk and Time • Positive Market Return • Risk reduction over time Topic 8 – Systematic Risk • Stocks carry systematic risk • Extension: Measure systematic risk with beta, compensated with CAPM formula	<u>S1 – The Role of Government</u> • Government and the financial life cycle • Education • Student Loans • Social Security • Healthcare • Housing • Tax and Government Debt <u>S2 – Trigonometry and Complex Numbers</u> • Trig on the unit circle • Complex numbers and 'imaginary interest' • Complex numbers on the 2d plane • Applications to the business cycle • Euler's formula <u>S3 – Advanced Mathematical Extensions</u> • Piecewise and step functions • Limits • Proofs • Conditional probability and expected value • Random Variables • Regression analysis

Unit 1: Financial Statements Unit Plan

Students must understand their financial capacities and how to precisely measure a financial situation in order to meet their financial goals.

This unit teaches students to understand financial matters in terms of wealth, rather than focusing only on cash flow. Students learn how to measure current wealth and changes to it using financial statements. Finally, they learn how to apply these tools to budgeting decisions that help them reach their goals.

Section 1: Goals for Unit & Desired Results

Essential Questions

Finance

- Why should I think about my financial situation in terms of wealth rather than cash?
- Which aspects of my financial situation are described by an income statement, a balance sheet, and a sources and uses statement?
- How can I use financial statements to create strategies to meet my financial goals?

Math

- How does manipulating equations and using functions help me model my financial situation more effectively?
- How can I use a spreadsheet to create and manipulate financial statements?

Unit Content

Knowledge (Students will know…)	**Skills** (Students will be able to…)
Financial Knowledge: 1. Financial transactions can be classified as income, expenses, assets and liabilities 2. Income Equation: Net Income = Gross income – expenses 3. Accounting equation: Net worth = assets – liabilities 4. T-Accounts can be used to represent financial transactions. 5. There are three components of a financial statement, each one providing a different kind of information: (i) income statement, (ii) balance sheet, (iii) and sources and uses 6. Financial decision making requires identifying goals and making tradeoffs 7. The importance of creating a budget and maintaining liquidity 8. Tax is an expense which must be accounted for	*Financial Skills:* 1. Distinguish between financial transactions and classify them as income, expenses, assets and liabilities 2. Select when it is appropriate and use the Income Equation: Net Income = Gross income – expenses 3. Select when appropriate and use the Accounting equation: Net worth = assets – liabilities 4. Use T-Accounts to represent financial transactions 5. Model a financial situation using a financial statement 6. Use financial statements to make financial decisions by identifying goals and making tradeoffs 7. Create a reasonable budget 8. Calculate taxes

Mathematical Knowledge:	*Mathematical Skills:*
1. Equations can be manipulated to isolate given variables 2. Variables can stand for not yet determined 3. Percent means "out of a 100" and percentages can be represented in a number of ways 4. Spreadsheets are a powerful tool for complex calculations, modeling financial situations and representing functions. 5. Calculating taxes involves averages and piecewise functions	1. Manipulate equations with multiple variables ('solving for x') 2. Substitute values for variables 3. Perform various operations related to working with percentages 4. Use mathematical formulas and functions in spreadsheets (SUM, +, -) 5. Use piecewise functions to calculate tax paid, marginal tax rate, and average tax rate

Standards

Math: CCSS	**Jumpstart Skills**
• A-CED.1 • A-CED.2 • A-CED.4 • A-REI.1 • A-REI.10 • A-SSE.1	• Spending and Saving: Standards 1 and 4 • Credit and Debit: Standards 1, 2 and 3 • Investing: Standards 1, 2, and 3 • Financial Decision Making: Standards 2, 4, 5 and 8

Section 2: Assessment Plan

Formative Assessment

1. **Topic Quizzes** - The quizzes will test the students' understanding of the key components of each topic in the course. It will include questions on the following topics:
    1. Understanding of vocabulary (relevant terms found in the Unit 1 glossary)
    2. Classifying financial transactions
    3. Checking T-Accounts
    4. Filling in T-Accounts
    5. Creating T-Accounts
    6. Filling in Income Statements
    7. Creating Income Statements from T-Accounts
    8. Filling in Balance Sheets
    9. Creating Balance Sheets from T-Accounts
    10. Filling in Sources and Uses Statements
    11. Creating Sources and Uses Statements from T-Accounts

12. Evaluating financial statements
13. Calculating taxes

2. **Student workbook** – In addition to the topic quizzes, the activities presented in the student workbook provide countless opportunities to check for student understanding by the teacher, either through collecting and grading work or through in-class observation. Workbook activities are intended to be used as needed as opportunities for formative assessment. This includes all spreadsheet work that the workbook directs students to complete.

Summative Assessment:

1. **Unit Project** - The take home project will require the students to look in detail at a complex financial scenario. They must create financial statements for a character and evaluate the situation to come up with a recommended financial strategy

2. **Unit Test** – The end of the unit exam will test students understanding of the topics covered in the unit and will cover all the material covered by the topic quizzes.

Section 3: Learning Plan

Scope and Sequence

Topic 1 – Understanding Wealth
a) Wealth and consumption
b) Introduction to concepts: income, expenses, assets and liabilities
c) Advanced topics: appreciation and depreciation: problem cases

Topic 2 – T Accounts
a) Introduction to T-Accounts: Simple examples with source and use of cash
b) Learn to balance transactions in the tables
c) T-Accounts: General examples with tables for Cash, Other Assets, Liabilities, Income, Expenses
d) T-Accounts comprehension: relate to character stories

Topic 3 – Income Statements
a) Introduce income equation, recap linear equations
b) Introduction to excel: creating tables and basic arithmetic functions
c) Creating income statements given figures
d) Connecting income statements and T-Accounts
e) Creating T-Account and income statements from character stories

Topic 4 – Balance Sheets

a) Introduce accounting equation and simple balance sheets
b) Create complex balance sheets
c) T-Accounts and beginning and ending balance sheets
d) Creating balance sheets from character stories

Topic 5 – Sources and Uses Statements

a) Introduction to liquidity: definition, examples and significance
b) Creating sources and uses statements
c) Relate sources and uses to T-Accounts and character stories

Topic 6 – Budgeting

a) Introduce budgeting: relate to net worth and net income and discuss strategies for budgeting
b) Explore financial decision making: weighing objectives and making trade-offs
c) Create budgeting strategies from financial statements; internet research

Topic 7 – Tax

a) Introduce tax – show place on financial statements
b) Fill out 1040
c) Tax brackets for income tax: marginal tax, average tax
d) Graphing tax against income
e) Modelling tax rates with piece wise functions

Unit 2: Earning Interest Unit Plan

Over a financial life cycle, the income you receive at a given time will not always match what you need to consume. You can transfer money either forward or backward so you can consume it when you need it. You transfer money forward by investing and transfer it backward by borrowing.

This unit teaches students the role of interest in loans and investment. It covers math related to exponents, Euler's number (*e*) and logs, which is necessary for students to make interest calculations. Students also learn about the financial instruments that are used to transfer consumption, and how they can help people achieve their financial goals.

Section 1: Goals for Unit

Essential Questions

Finance

- What is the role of interest income and interest expenses in the financial life cycle?
- Why is a dollar received in the future worth less than a dollar now?
- What financial tools allow me to transfer money forward and backward most effectively?

Math

- How does math allow me to calculate the most effective strategies for transferring money forward and backward?
- Why do interest calculations involve exponents, and why is this significant?

Unit Content

Knowledge (Students will know…)	**Skills** (Students will be able to…)
Financial Knowledge: 1. Investments and loans allow you to transfer consumption across time 2. The formulas for calculating interest: Simple Interest Formula, Compound Interest Formula, Continuous Compounding Formula 3. Why the rule of 72 works and why it is used 4. There are many different financial instruments that allow for different ways of transferring consumption with different costs/benefits 5. The importance of your credit score when transferring consumption	*Financial Skills:* 1. Calculate present value and future value in situations involving investments and loans 2. Select and use the appropriate formulas for calculating interest in various contexts 3. Apply the rule of 72 and identify situations when it is appropriate to estimate using the rule of 72 4. Create investment and borrowing plans for characters based on their financial situation 5. Identify factors that positively/negatively affect one's credit score

Mathematical Knowledge: 1. Exponentiation is repeat multiplication 2. Euler's number, *e*, is an irrational number, which can be defined as a limit 3. Logs are functions whose outputs are exponents 4. Functions can be broken down into smaller pieces or components and programmed into a spreadsheet 5. A sum of a series can be calculated using either a formula or a spreadsheet 6. The difference between linear and exponential growth 7. Extension: Fractional powers are the same as nth roots, and the inverse of integer powers 8. Extension: Limits can be used to interpret functions at various values	*Mathematical Skills:* 1. Manipulating and calculating formulas with exponents 2. Calculating values of equations using Euler's number: *e* 3. Using logs to solve equations involving a variable in the exponent 4. Develop strategies to break down complex functions and input them in a spreadsheet 5. Calculate the sum of series using spreadsheet software 6. Sketch and interpret graphs displaying linear and exponential growth 7. Extension: Manipulate and calculate expressions containing fractional powers 8. Extension: Calculate and interpret the limits of a range of functions

Math: CCSS	**Jumpstart Skills**
• A-CED.1 • A-CED.2 • A-CED.4 • A-REI.1 • A-REI.10 • A-SSE.1	• Spending and Saving: Standards 1 and 4 • Credit and Debit: Standards 1, 2 and 3 • Investing: Standards 1, 2, and 3 • Financial Decision Making: Standards 2, 4, 5 and 8

Section 2: Assessment Plan

Formative Assessment

1. **Topic Quizzes** - The quizzes will test the students' understanding of the key components of each topic in the course. It will include questions on the following topics:
    1. Understanding of key concepts
    2. Calculating simple interest
    3. Calculating compounding
    4. Calculating compound interest
    5. Calculating continuously compounded interest (Move between simple interest and continuously compounded interest)

6. Rule of 72
7. Financial instruments: savings account, APR, CD account, money market account, bonds, credit cards, student loans, bank loans
8. Credit Scores
9. Multi-part question: Calculating total earned combining multiple investments and loans
10. Evaluative questions: looking at different options for investment, calculating which earns most over a period
11. Use interest calculations to create financial statements
12. Character story questions

2. **Student workbook** – In addition to the topic quizzes, the activities presented in the student workbook provide countless opportunities to check for student understanding by the teacher, either through collecting and grading work or through in-class observation. Workbook activities are intended to be used as needed as opportunities for formative assessment. This includes all spreadsheet work that the workbook directs students to complete.

Summative Assessment:

1. **Unit Project** - The take home project will require the students to look in detail at a complex financial scenario. They must create financial statements for a character and evaluate the situation to come up with a recommended financial strategy.

2. **Unit Test** – The end of the unit exam will test students' understanding of the topics covered in the unit and will cover all the material covered by the topic quizzes.

Section 3: Learning Plan

Scope and Sequence

Topic 1 – Introduction to Earning Interest (about 1 day of content)

a) Discuss transferring consumption across time: Investing and borrowing
b) Introduce time value of money
c) Define key concepts: Present value, future value, interest rate, period of interest

Topic 2 – Mathematics of interest

a) Calculate simple interest
b) Introduce compounding
c) Introduce equation: $FV = PV(1 + r)^n$
d) Recap/Remediate the rules of exponents
e) Comparing simple and compound interest: linear and exponential growth

Topic 3 – Compound Interest

a) Explore receiving interest payments within a year
b) Explain the principle of compounding within a year

c) Use compound interest formula: FV=PV(1+r/n)^n
d) Explain concept of continuously compounded interest
e) Introduce e (Euler's number): e=lim (1+1/n)^n
f) Use continuous compounding formula: FV=PVe^r
g) Extension: Explore limits more rigorously

Topic 4 – The Rule of 72 – Double your money

a) Introduce logs
b) Show how to convert continuously compounding to simple interest
c) Introduce rule of 72
d) Use the rule of 72 to make appropriate estimates
e) Extension: Derive the rule
f) Extension: Convert other forms of compounding using fractional powers

Topic 5 – Present Value

a) Explain Accelerating Consumption and borrowing
b) Define discounting
c) Derive discounting formula: PV=FV/(1+1/n)^n [equivalent: PV=FV(1+1/n)^-n]
d) Apply the rule of 72

Topic 6: Earning Interest in the Financial Life Cycle

a) Explain that different financial instruments allow for different ways of transferring consumption with different costs/benefits
b) Discuss different tools for investment: savings account, CD, MMA, bonds, stocks
c) Discuss different tools for borrowing: bank loans, student loans
d) Look in detail at features of credit cards
e) Identify the right financial instrument for different financial needs
f) Incorporate interest calculations in financial statements.

Topic 7 – Credit Scores

a) Explain the role of credit scores
b) Explain how scores are calculated
c) Discuss measures to improve your credit score

Unit 3: Regular Payments Unit Plan

Unit 2 discussed transactions in which we invest or borrow a given amount of money at one point in time and reclaim or repay it in full at a later time. Often, though, we receive returns or make repayments in *installments*. For example: we usually make monthly payments on a mortgage. When we have regular payments, calculating interest becomes more complex.

This unit gives students an understanding of regular payments. Students learn to calculate arithmetic and geometric series and apply this math to payment series. They learn about the financial problems that require regular payments, from retirement to home-owning, and the financial instruments that are used. Students learn how to model payment series in spreadsheets, which are ideally suited to such calculations.

Section 1: Goals for Unit

Essential Questions

Finance

- Why can I model regular payments as geometric series?
- What financial tools allow me to use payment series most effectively?

Math

- How does math allow me to calculate the most effective strategies for investing and borrowing with payment series?
- How can a spreadsheet break down complex cash flows?

Unit Content

Knowledge (Students will know…)	**Skills** (Students will be able to…)
Financial Knowledge: 9. Constant payments and growing cash flows 10. Mortgage payments and retirement savings 11. The relationship between inflation and future value	*Financial Skills:* 9. Calculate present and future value of constant payments and growing cash flows 10. Apply payment series calculations to mortgage payments and retirement savings 11. Take into account inflation to calculate real present or future value

Mathematical Knowledge: 9. Sigma notation can be used to represent various kinds of sums 10. The terms in an Arithmetic Series have a common difference 11. The terms in a Geometric Series have a common ratio 12. Use spreadsheet software to model constant payments and growing cash flows 13. The concept of a limit	*Mathematical Skills:* 6. Represent various series and sums using Sigma Notation 7. Calculate the *n*th term for any arithmetic or geometric series 8. Sum any arithmetic or geometric series 9. Decompose complex series in a spreadsheet 10. Use limits to calculate the value of perpetuities

Standards

Math: CCSS	Jumpstart Skills
• A-APR.1 • A-CED.2 • A-REI.1 • A-REI.10 • A-SSE.1 • A-SSE.2 • A-SSE.4 • F-BF.1 • F-BF.2 • F-LE.2 • F-LE.5	• Spending and Saving: Standards 1 and 4 • Credit and Debit: Standard 3 • Employment and Income: Standards 2 and 3 • Financial Decision Making: Standards 1, 2, 4, 5 and 8

Section 2: Assessment Plan

Formative Assessment

1) **Topic Checks for Understanding** - The Checks for Understanding will test the students' understanding of the key components of each topic in the course. It will include questions on the following topics:
    1. Arithmetic Series
    2. Geometric Series
    3. Annuities
    4. Mortgages
    5. Debt to income ratio
    6. Down payment

7. Growing cash flow
8. Growing Payments
9. Dividend discount model
10. Inflation
11. Financial instruments for payment series, and consumer strategies for using them effectively
12. Saving for retirement
13. Use payment series calculations to create financial statements in spreadsheets

2. **Student workbook** – In addition to the topic quizzes, the activities presented in the student workbook provide many opportunities to check for student understanding by the teacher either through collecting and grading work or through in-class observation. Workbook activities are intended to be used as needed as opportunities for formative assessment. This includes all spreadsheet work that the workbook directs students to complete.

Summative Assessment:

3. **Unit Project** - The take home project will require the students to look in detail at a complex financial scenario. They must create financial statements for a character and evaluate the situation to come up with a recommended financial strategy
    - Note that for students taking the abridged version of this unit, the 'Simone' project can be completed using only the materials from topics 1-3. It does require knowledge of the optional extension to topic 3 concerning car purchases.

4. **Quizzes** – These cover the same material as the workbook checks for understanding and they can be used at the end the of each topic, or can be the source material for a single end of unit exam. They can be found in the 'Unit 3 Quizzes' folder.

Section 3: Learning Plan

Scope and Sequence

Topic 1 – Arithmetic Series
a) Arithmetic sequences and series
b) Sigma notation and indices
c) Creating and summing arithmetic series in spreadsheets

Topic 2 – Geometric Series
a) Geometric sequences and series
b) Creating and summing series in spreadsheets
c) Extension: Deriving the geometric series formula

Topic 3 – Annuities
a) Introduce concept of saving/paying off loans in installments

b) Calculate future value of annuity
c) Calculate present value of annuity
d) Understand difference in calculation when payments are made at beginning and end of period
e) Modelling annuities in spreadsheets
f) Extension: Deriving the annuity formula

Topic 4 – Mortgages

a) How to calculate mortgage value
b) Advantages/disadvantages of a mortgage
c) Creating amortization schedules in spreadsheets
d) (Extension) Car Purchase

Topic 5 –Growing cash flow

a) Examples of growing payments. Stock dividends, saving % of income
b) Apply geometric sequences to calculation of growing cash flows
c) Model growing cash flows in spreadsheet
d) Perpetuities and Dividend discount model (d/r-g)
e) Extension: Derive equations

Topic 6 – Retirement

a) Concept of retirement in relation to the financial life cycle
b) Calculating how much is needed for retirement
c) Using spreadsheet to model savings value for retirement
d) Calculating how much to save for retirement

Topic 7 – Interest Rates and Inflation

a) Show how Inflation acts like a negative growth rate, decreasing the value of future cash
b) Calculating inflation adjusted future value
c) Consumer Strategies with debt and investment

Unit 4: Expected Value and Insurance Unit Plan

The future isn't certain—and this applies to your financial situation as much as anything else. You could get a lucrative promotion, or you could be fired. You could maintain perfect health, or you could require expensive medical procedures. Your investments could increase in value, or they could decrease. Risk concerns the potential for undesirable outcomes such as these.

In this unit, students learn how to model and manage financial risk. They learn about the financial tools that are needed, in particular the central types of insurance. Students are also taught the mathematics required to evaluate risk: probability, expected value and the binomial theorem.

Section 1: Goals for Unit

Essential Questions

Finance

- How does risk affect my financial plans?
- How does insurance allow me to manage risk?

Math

- How does math help me evaluate financial risk?
- Why is diversified risk better than stubborn risk?

Unit Content

Knowledge (Students will know…)	**Skills** (Students will be able to…)
Financial Knowledge: 1. There are different kinds of financial risk 2. There are different kinds of insurance, each with their own merits and drawbacks 3. Expected Utility allows us to calculate an expected outcome in situations involving non-numerical factors	*Financial Skills:* 1. Distinguish between diversified and stubborn risk in different scenarios 2. Use mathematics to evaluating insurance purchases 3. Use expected utility to model and analyze uncertain situations
Mathematical Knowledge: 1. Probability is a measure of how likely an event is 2. Expected Value is the average payout/outcome in uncertain situations 3. What makes an experiment a binomial experiment and how the binomial theorem can be used to understand such situations	*Mathematical Skills:* 1. Calculate probability in various situations 2. Analyze and model uncertain situations using expected value 3. Apply the binomial theorem to calculate diversified risk

Standards

Math: CCSS		Jumpstart Skills
• S-CP.1 • S-CP.2 • S-CP.3 • S-CP.4 • S-CP.5 • S-CP.6 • S-CP.7 • S-CP.8 • S-CP.9	• S-MD.1 • S-MD.2 • S-MD.3 • S-MD.4 • S-MD.5 • S-MD.6 • S-MD.7	• Risk Management and Insurance: Standards 1, 2 and 3 • Financial Decision Making: Standards 2, 4, 5 and 8

Section 2: Assessment Plan

Formative Assessment

1. **Topic Quizzes** - The quizzes will test the students' understanding of the key components of each topic in the course. It will include questions on the following topics:
    1. Key concepts related to financial risk and insurance
    2. Calculating cost of insurance
    3. Expected value with financial uncertainty
    4. Expected value of insurance purchase
    5. Expected utility when facing financial risk
    6. Identifying diversified risk and stubborn risk
    7. Calculating diversified risk with binomial theorem

2. **Student workbook** – In addition to the topic quizzes, the activities presented in the student workbook provide countless opportunities to check for student understanding by the teacher, either through collecting and grading work or through in-class observation. Workbook activities are intended to be used as needed as opportunities for formative assessment. This includes all spreadsheet work that the workbook directs students to complete.

Summative Assessment:

1. **Unit Project** - The take home project will require the students to look in detail at a complex financial scenario. They must create financial statements for a character and evaluate the situation to come up with a recommended financial strategy

2. **Unit Test** – The end of the unit exam will test students' understanding of the topics covered in the unit and will cover all the material covered by the topic quizzes.

Section 3: Learning Plan

Scope and Sequence

Topic 1 – Introduction to insurance

a) Risk in the financial life-cycle
b) Insurance as a tool for avoiding risk (health, vehicle, property, life)
c) Types of insurance (deductibles and copays)

Topic 2 – Probability

a) Compound experiments
b) Independent events
c) Tree diagrams

Topic 3 - Expected Value

a) Expected value and decision making

Topic 3 – Expected Value and Insurance

a) Calculating EV of insurance
b) Expected value vs. expected utility
c) Comparing EV and EU for insurance

Topic 4 – Diversifying risk

a) Definitions of diversified and systematic risk
b) Dice game
c) Applications to insurance

Topic 5 – The Binomial theorem

a) Introducing experiments and trials
b) Binomial trees
c) Choice function
d) Binomial theorem
e) Applications to insurance

Unit 5: Risk and the Stock Market Unit Plan

When saving money over the long term, one of the most effective ways to earn a high rate of return is by investing in the stock market. Investing in stocks can come with varying levels of risk, and various financial instruments allow for managing this risk.

In this unit, students are introduced to the stock market and the key concepts required to invest in it. They learn the probability and statistics required to calculate risk associated with stocks. They will also understand how different financial tools help manage risk through diversification.

Section 1: Goals for Unit

Essential Questions

Finance

- Why does investing in stocks require taking on risk?
- How does the efficient market hypothesis imply you should approach stock market risk?

Math

- What is the relationship between random walks and normal distributions?
- How do the formulas for mean and standard deviation affect the relationship between risk and time?

Unit Content

Knowledge (Students will know…)	**Skills** (Students will be able to…)
Financial Knowledge: 1. How and why prices change in the stock market 2. The efficient market hypothesis and why a stock price evolves as a random walk 3. Portfolios can be used as a tool for diversifying risk 4. The risk of investing in the market decreases the longer you stay in the market 5. Systematic risk is risk the whole market faces 6. There are various kinds of financial instruments for stock market investment 7. Capital asset pricing model	*Financial Skills:* 1. Explain why prices change in the stock market 2. Model the price of a stock over time and calculate the probability of different possible outcomes in price movement 3. Select financial instruments that help diversify risk for a given portfolio 4. Show, using mathematics, how holding a stock over a longer period reduces risk 5. Identify systematic risk in the market 6. Evaluate scenarios to decide which financial instrument is most appropriate 7. Use the CAPM model to relate risk level and expected return

Mathematical Knowledge: 1. Understand what makes something a Random Walk 2. Identify binomial distributions 3. Normal distribution and standard deviation	*Mathematical Skills:* 1. Can calculate the probability of various outcomes of a random walk 2. Model stock activity using binomial distributions 3. Use standard deviation to calculate the risk level of a portfolio

Skills

Math: CCSS		Jumpstart Skills
• S-IC.1 • S-IC.2 • S-IC.3 • S-IC.4 • S-IC.5 • S-IC.6 • S-IC.7 • S-IC.8 • S-IC.9	• S-MD.1 • S-MD.2 • S-MD.3 • S-MD.4 • S-MD.5 • S-MD.6 • S-MD.7	• Spending and Saving: Standards 1 and 4 • Investing: Standards 1, 2 and 3 • Financial Decision Making: Standards 2, 4, 5 and 8

Section 2: Assessment Plan

Formative Assessment

1. **Topic Quizzes** - The quizzes will test the students' understanding of the key components of each topic in the course. It will include questions on the following topics:
    1. Key concepts related to the stock market
    2. Calculating expected return of stock
    3. Understanding of how information affects stock price
    4. Understanding of the efficient market hypothesis
    5. Calculate expected portfolio return
    6. Calculate risk for portfolios
    7. Calculate portfolio risk and expected return over time
    8. Understand systematic risk
    9. Evaluate which stock market investment instruments are best suited to particular situations

2. **Student workbook** – In addition to the topic quizzes, the activities presented in the student workbook provide countless opportunities to check for student understanding by the teacher, either through collecting and grading work or through in-class observation. Workbook activities are intended to be used as needed as opportunities for formative

assessment. This includes all spreadsheet work that the workbook directs students to complete.

Summative Assessment:

1. **Unit Project** - The take home project will require the students to look in detail at a complex financial scenario. They must create financial statements for a character and evaluate the situation to come up with a recommended financial strategy

2. **Unit Test** – The end of the unit exam will test students' understanding of the topics covered in the unit and will cover all the material covered by the topic quizzes.

Section 3: Learning Plan

Scope and Sequence

Topic 1 – Introduction to Stocks

a) Definition of stock; equity
b) Income from stocks: capital gain, dividend
c) Expected value of stocks

Topic 2 – The efficient market hypothesis

a) New information affects share price
b) How share price is determined
c) Over-valued and under-valued stocks
d) The efficient market hypothesis

Topic 3: Random Walks

a) Introduce random walks
b) Efficient market hypothesis implies stock prices are random walks
c) Random walk games
d) Model stock price movement

Topic 4 – Normal distribution and standard deviation

a) Introduce standard deviation
b) Calculate population and sample standard deviation on calculator/excel
c) Normal distribution

Topic 5 – Normal distribution and binomial distribution

a) Binomial distributions approximate normal distribution
b) Mean and SD for binomial distributions
c) Applications for insurance

Topic 6 – Risk and portfolios

a) Expected return and standard deviation of shares

b) Expected return and standard deviation of portfolios
c) Portfolios reduce risk through diversification

Topic 7 – Systematic Risk

a) Stocks carry systematic risk
b) Extension: Measure systematic risk with beta
c) Extension: Risk compensated with CAPM formula

Supplemental Unit 1: The Role of Government

An individual is part of a wider society, and their personal finance is part of a wider economy. People's needs across different generations meet within this economy – with the savers lending to the borrowers. This interaction is supplemented by government-run social programs.

In this unit, students will learn about government programs that take on systematic risk and transfer consumption across generations. They will research these tax-funded programs, and they will compare programs in the US with those across the world.

This unit will be almost entirely project focused. Students will research and compare government programs in the US with other countries, and look at how these influence an individual's decision when navigating the financial life cycle.

Topic Covered

1. Government and the financial life cycle
2. Education
3. Student Loans
4. Social Security
5. Healthcare
6. Housing
7. Tax and Government Debt

Supplemental Unit 2: Trigonometry and Complex Numbers

In this unit, mathematical applications of finance are extended to cover topics in trigonometry and complex numbers. Through an analogy to the concept of continuously compounded interest, students gain an understanding of complex numbers and trigonometric functions on the unit circle. This unit focuses on a geometric interpretation of complex numbers, and thinking of sin and cosine on the unit circle.

Topics Covered

1. Review Continuously Compound interest: Students will use the rules of exponents to use and understand the compound interest formula FV = PV $(1+r/n)^{nt}$. Students will use limits, and the definition of Euler's Number to derive the continuous compounding formula.
2. Understanding Complex numbers) Complex numbers will be presented as representing two different dimensions (two different currencies.) Students will add, subtract and multiply complex numbers. They will explore the complex analog of interest. The focus will be on the operation of multiplying a complex number by (1 + r*i*). They will graph the results of this operation and see that this produces a change perpendicular to the original complex number.
3. The unit circle: Students will look at $\lim_{n\to\infty}\left(1+\frac{ri}{n}\right)^{n}$ and see that this creates an arc on the unit circle.
4. Euler's formula: Students will learn that $e^{i\pi}$ is a 180 degree rotation and therefore transforms 1 into -1. This leads to Euler's formula: $\boldsymbol{e^{i\pi}+1=0}$ which at first seems magical, but gradually students gain an understanding of the relationship between $\boldsymbol{e}$, $\boldsymbol{\pi}$ $\boldsymbol{and}$ $\boldsymbol{i}$ **.**
5. Trigonometric Functions: Using the formula $a^2 + b^2 = 1$ for points on the unit circle, and $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, students explore trigonometric relationships on the unit circle.
6. Business Cycle: Students learn how sine and cosine produce cyclical patterns that are similar to those observed in the Business Cycle.

Supplemental Unit 3: Advanced Extensions

In this unit, students investigate more advanced mathematical models of the financial topics discussed in the previous five units. This may provide advanced students with a strong foundation to go on to a first course in calculus.

Topics Covered

1. Piecewise and step functions
2. Limits
3. Proofs
4. Conditional probability and expected value
5. Random Variables
6. Regression analysis