

MATHEMATICS OF TAX-ADVANTAGED RETIREMENT SAVINGS – PART 2



1. In our first lesson on Tax-Advantaged Retirement Savings Accounts we assumed a constant tax rate across our entire life. What could lead to a person paying a different tax rate at different times in the life cycle?

Changing Tax Rates

When we assumed constant tax rates, the Traditional IRA (pre-tax money) and the Roth IRA (post-tax money) had the same return. All that differed was the timing of the taxes. If your tax rates when working and when retired are different then this is no longer the case.

- In life, tax rates are not likely to be constant. Depending on your income at a given phase of your career and your household make-up, your tax rates will likely change and could change significantly.
- Additionally, tax rates change over time due to changes in government policies.

Tax Rates and Income

The following table shows tax rates for 2024.

Table 1. 2024 Federal Income Tax Brackets and Rates for Single Filers, Married Couples Filing Jointly, and Heads of Households

Tax Rate	For Single Filers	For Married Individuals Filing Joint Returns	For Heads of Households
10%	\$0 to \$11,600	\$0 to \$23,200	\$0 to \$16,550
12%	\$11,600 to \$47,150	\$23,200 to \$94,300	\$16,550 to \$63,100
22%	\$47,150 to \$100,525	\$94,300 to \$201,050	\$63,100 to \$100,500
24%	\$100,525 to \$191,950	\$201,050 to \$383,900	\$100,500 to \$191,950
32%	\$191,950 to \$243,725	\$383,900 to \$487,450	\$191,950 to \$243,700
35%	\$243,725 to \$609,350	\$487,450 to \$731,200	\$243,700 to \$609,350
37%	\$609,350 or more	\$731,200 or more	\$609,350 or more

Source: [2024 Tax Brackets and Federal Income Tax Rates | Tax Foundation](#)

Generally, the higher your income, the higher your tax rate

- Note, the table gives **marginal** tax rates. This means you pay the higher rate only on the dollars earned above the cutoff point.

Example: You are a single filer with a (taxable) income of \$60,000. Let's breakdown your income tax using the table.

- Your income is in the \$47,150-\$100,525 bracket so your marginal tax rate is 22%. So, for each additional dollar you earn, you will have to pay 22 cents in tax.
- Your first \$11,600 of income is taxed at 10%, which amounts to \$1160.
- Your income from \$11,600 to \$47,150 is taxed at 12%, giving you $0.12 * (47150 - 1160) = 5515.80$.
- Your income from \$47,150 to \$60,000 is taxed at 22%, giving you $0.22 * (60000 - 47150) = 2827$.
- So, your total tax is \$9502.80, and your average tax rate is $\frac{9502.8}{60000} = 0.16$ or 16%.

For more on tax rates, see Unit1 Topic 7 from the FiCycle Curriculum.



2. Suppose you are married, filing jointly and have a taxable income of \$250,000. What is your marginal tax rate, income tax paid and average tax rate?

We can think of one's financial life as having 5 stages:

Education, Early Career, Mid-Career, Later Career, and Retirement.

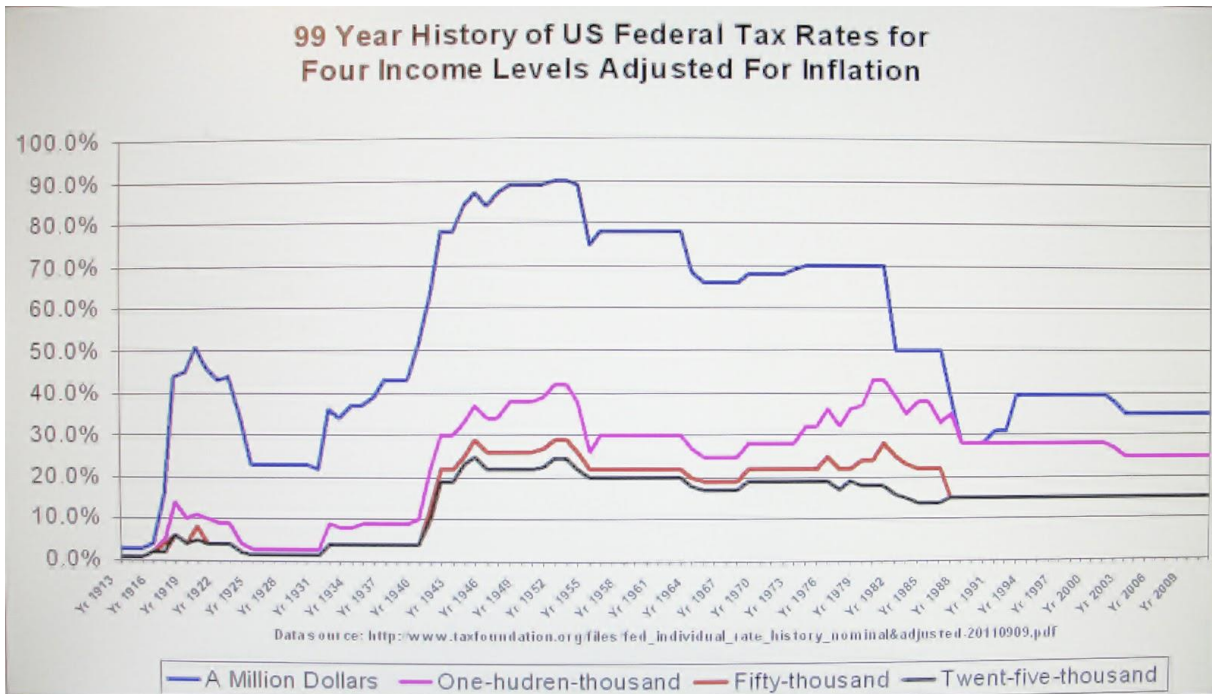


3. Describe a circumstance in which a person's tax rate might be **lower** at a particular phase of their life than in retirement.
4. Describe a circumstance in which a person's tax rate might be **higher** at a particular phase of their life than in retirement.

For many people, taxes are lowest when they are in school and remain relatively low when they first start working and then rise over time as their income increases and then decline in retirement when their income declines; however, this not always the case. During working years and retirement there could be years of much higher or lower income and tax rates.

Tax rates also change over time

While individuals experience different tax rates as their income varies, the government has also changed the tax rate numerous times throughout history. The following chart shows tax rates at various levels of income on an inflation adjusted basis (adjusted to 1982). While the tax rates for the highest earners vary significantly, the tax rates on middle- and lower-income people have for the past 80 years remained in a narrower range.



[Data-Driven Viewpoints: A 99 YEAR HISTORY OF TAX RATES IN AMERICA \(aseyeseesit.blogspot.com\)](http://www.aseyeseesit.blogspot.com)

Retirement Savings Calculations

As we can see, your income level affects your tax rate, which determines whether a Roth or Traditional IRA will leave you with more money.

Example: Suppose you are depositing \$10,000 in an IRA, in an investment that will triple in value between now and when you retire.

- Your marginal tax rate now is 22% and the tax you will pay on retirement income is 12%
- In a Roth IRA, this will give you $(1 - 0.22) * 10000 * 3 = 23400$ in after-tax retirement income.
- In a traditional IRA, this will give you $10000 * 3 * (1 - 0.12) = 26,400$ in after-tax retirement income – in other words, you end up with an additional \$2000

We can represent this algebraically by revising the equations from the previous lesson. This time we will need different variables for the various tax rates. These are reflected by having subscripts for tax rates during each period:

- τ_w tax rate when working
- τ_r tax rate during retirement
- r pre-tax rate of return on investment

The equations for returns on each these accounts also need to be updated to incorporate changing tax rates:

- After tax return on Traditional IRA investments: $(1 - \tau_r)(1 + r)^T$
- After tax-return on Roth IRA investments: $(1 - \tau_w)(1 + r)^T$

Both have the same return so the relative performance is a result only of the tax rates. The ratio of Traditional to Roth IRA performance is then:

$$\frac{(1 - \tau_r)(1 + r)^T}{(1 - \tau_w)(1 + r)^T} = \frac{(1 - \tau_r)\cancel{(1 + r)^T}}{(1 - \tau_w)\cancel{(1 + r)^T}} = \frac{(1 - \tau_r)}{(1 - \tau_w)}$$

Since tax rates are percentages (that is between zero and 1), both the numerator and denominator must be positive.

- The ratio will be *greater than one* when the tax rate at retirement is smaller than the tax rate when working (at the time the contribution to the account is made).
→ If the ratio is *greater than one* the traditional IRA is outperforming the Roth IRA
- The ratio will be *less than one* when the tax rate at retirement is higher than the tax rate when working (at the time the contribution to the account is made).
→ If the ratio is *less than one* the Roth IRA is outperforming the traditional IRA

Thinking through an example

Suppose your marginal tax rate now is 12% and you expect it to be at 22% when you retire. Which produces a higher after-tax return: A traditional IRA or a Roth IRA? By how much?

Since the current tax rate is lower, a Roth IRA will produce a higher return:

The comparison ratio of Traditional IRA to Roth IRA is:

$$\frac{1 - 22\%}{1 - 12\%} = 88.6\%$$

Even though the tax rate almost doubles, the effect on return is only an 11.4% reduction.

- We saw in part one of this lesson that the difference in value between a taxable account and an IRA can be much more significant, the IRA may end up double (or more) of the value of the taxable account.
 - The value of a taxable account is $(1 - \tau_w)(1 + r * (1 - \tau_i))^T$ where τ_i is the tax on interest income. This is equivalent to $(1 - \tau_w)(1 + r)^T * (1 - \tau_i)^T$ in other words, the tax on interest is applied T times while the tax on income is only applied once.
 - This means that avoiding the interest tax has a much bigger effect than whether the tax on income comes at the working or retirement rate.
 - Thus, while it seems like a Roth IRA is significantly better, the math shows that it offers only a slight advantage over a Traditional IRA in this case, and in general the math tells us that it's much more important to invest in one retirement account or another than worrying about choosing between them.
5. You currently have a marginal tax rate of 18% and you expect it to be at 20% when you retire. Which produces a higher after-tax return: A traditional IRA or a Roth IRA? Use a ratio to compare the two and explain what your results mean.

 6. You have a marginal tax rate now is 33% and you expect it to be at 22% when you retire. Which produces a higher after-tax return: A traditional IRA or a Roth IRA? Use a ratio to compare the two and explain what your results mean.

 7. Suppose your marginal tax rate now is 25% and you expect it to be at 25% when you retire. Which produces a higher after-tax return: A traditional IRA or a Roth IRA? Use a ratio to compare the two and explain what your results mean.

The decision to set up a Roth IRA is more complicated than it seems at first and may rest on factors other than tax rate:

- *How long do you expect to be at your current income level and current tax bracket?*
This will affect how much money you will eventually have in the account. If you only invest in the Roth IRA for a few years, it may not build to a significant enough level for you to focus on investing it.
- *What type of investments do you plan to use?*
Investments that generate high levels of taxable income like bonds or active stock trading may make setting up an IRA more valuable.
- *Are you able to commit to saving money until retirement?*
Roth IRAs offer more alternatives for early withdrawals of money from the retirement account without penalties. Note that you will still need to pay taxes on earnings at the time of withdrawal.