MATHEMATICS OF TAX-ADVANTAGED RETIREMENT SAVINGS – PART 1

ANSWERS

1. There are a lot of possible answers. Retirement savings, social securty, a pension, relying on one's children or family.

2. $\$7{,}000(1-0.25)=\$5{,}250$

Taxable Investment:	Individual Retirement Account (IRA):
$P(1-\tau)\big(1+r(1-\tau)\big)^T$ $= \$7{,}000(1-0.25)\big(1+0.08(1-0.25)\big)^{36}$ $= \$42{,}773.07$ $5{,}250 \bullet x = 42{,}773.07$ $x = 8.147$ → Approximately an 8x multiplier	$P(1-\tau)(1+r)^T$ $= \$7{,}000(1-0.25)(1+0.08)^{36}$ $= \$\,83{,}832.90$ $5{,}250 \bullet x = 83{,}832.90$ $x = 15.968$ → Approximately a 16x multiplier

This makes sense with the rule of 72. Sine the IRA doubles one additional time more than the taxable invetment. We can see this in how the multiplier is also double.

For questions 3-5, suppose you invest $5,000 at 7.2% annual interest for 30 years and you have a 33% tax rate. You can choose to invest in a taxable investment account or an IRA.

3. How much money would you have with a taxable investment vs. IRA when you withdraw the money during retirement?

Taxable Investment:	Individual Retirement Account (IRA):
$P(1-\tau)\big(1+r(1-\tau)\big)^T$ $= \$5{,}000(1-0.33)\big(1+0.072(1-0.33)\big)^{30}$ $= \$13{,}767.87$	$P(1-\tau)(1+r)^T$ $= \$5{,}000(1-0.33)(1+\mathbf{0.072})^{30}$ $= \$26{,}970.46$

4. What is your investment return after taking taxes into account with a taxable investment? What is your investment return after taking taxes into account on an IRA?

Taxable Investment:	Individual Retirement Account (IRA):
$P(1-\tau)\big(1+r(1-\tau)\big)^T$ $= \$5{,}000(1-0.33)\big(1+\mathbf{0.072}(\mathbf{1-0.33})\big)^{30}$ $= \$13{,}767.87$ $.072(1-0.33) = 0.072(0.67) = 0.048 = 4.8\%$	$P(1-\tau)(1+r)^T$ $= \$5{,}000(1-0.33)(1+\mathbf{0.072})^{30}$ $= \$26{,}970.46$ $0.072 = 7.2\%$

5. Using the values, you calculated in the previous question and the rule of 72, approximately how much would you expect to have if you put your money in a taxable investment? Approximately how much would you have if you put it in an IRA?

$\frac{72}{4.8} = 15$ An investment with a return of 4.8% will take approximately *15 years* to double in value.	$\frac{72}{7.2} = 10$ An investment with a return of 7.2% will take approximately *10 years* to double in value.
In 30 years it will double twice according to the rule of 72. $\$5{,}000(1 - 0.33) = \$3{,}350 \rightarrow$ $\$10{,}000(1 - 0.33) = \$6{,}700 \rightarrow$ $\$20{,}000(1 - 0.33) = \$13{,}400$	In 30 years it will double three time according to the rule of 72. $\$5{,}000(1 - 0.33) = \$3{,}350 \rightarrow$ $\$10{,}000(1 - 0.33) = \$6{,}700 \rightarrow$ $\$20{,}000(1 - 0.33) = \$13{,}400$ $\$40{,}000(1 - 0.33) = \$26{,}800$

As a result it makes sense that the IRA returns about double what the taxable investment returns since it will double one additional time.

6. The commutative property.

7. It is likely to change. As one enters retirement they are likely to expereince a dip in their tax rate as their income is likely to decrease when they stop working.

8. There are lots of possible answers. One possible answer: Yes, if our tax rate changes then it changes the math we did and might change the outcome.

9. $P(1 - \tau)(1 + r)^T$
$= \$6{,}000(1 - 0.19)(1 + \mathbf{0.085})^{25}$
$= \$37{,}357.67$

10. Yes the matching makes it very motivating to contribute more, as the company matching up to $5,000 in contributions is doubling your contribution (up till that amount) and it makes a large and positive effect on your return on investment. After $5,000 in contributions there is no further benefit with regards to the company matching and so it does not motivate any futhrer contributions beyond that amount.