FiCycle Standards for Personal Finance and Mathematics: Rationale

1. Our Principles

At Financial Life Cycle Education (FiCycle), we believe it is essential for students to receive a high-quality education in both personal finance and mathematics. Further, we believe that offering students a mathematics course that is based upon applications to finance advances both goals. In this document, we present the research and theory behind these beliefs and outline a set of standards for a course in personal finance. We are also responding to greater focus in the math education community on rigorous alternative pathways for high school math education, which calls for standards for financial mathematics that link the two subjects conceptually rather than independent mathematics and personal finance standards.

Our research-based approach rests on the following principles:

1. Students learn best when they develop conceptual understanding, and conceptual understanding is required to utilize financial knowledge as financial products and services evolve.
2. A conceptual understanding of personal finance requires understanding the underlying mathematics.
3. Students learn best when they find the material interesting and relevant to their lives.
4. Many students who are disengaged with pure mathematics find applications to finance relevant and interesting.
5. Student and teacher time is a scarce and valuable resource.

Principles 1-2 explain why we believe that teaching finance within a mathematics course improves the quality of finance education, which is especially needed during a time of rapid development of new financial technologies.

Principles 3-4 explain why we believe basing a mathematics course around applications to finance improves the quality of mathematics education. Principle 5 is in effect a cautionary note that guides our thinking: it says that introducing a new course into the curriculum can come with a significant opportunity cost, since another course must be removed from the students' schedule to make room. We believe that replacing a traditional third-year or fourth year mathematics course with a mathematical finance course is uniquely positioned to justify this cost, as students in this case retain the additional mathematics training while gaining additional benefits through the financial applications.

In what follows, we outline the justification for these principles.

2. Understanding Personal Finance

Research shows that young people across the US are in a precarious financial situation, and

many lack essential financial knowledge.[1] Further, it is known that this low financial knowledge is closely connected to undesirable financial outcomes.[2]

Financial education has been proposed as a solution to this pressing problem. Though such education programs have been shown, on the whole, to improve financial knowledge, their effectiveness on financial outcomes varies greatly. It is important, therefore, to better understand the best practices that lead to effective education programs.[3]

A key finding in education research is that effective learning requires conceptual understanding. Being taught a series of procedures for completing a list of rote tasks, without addressing the connections between them, leads students to learn little, and struggle to remember what they do learn. Students learn and retain knowledge much better when they grasp the fundamental concepts connecting the different areas.[4] Since one of the key goals of financial education is for students to retain what they learn and apply it to key financial decisions in adulthood, long term knowledge retention is particularly important for financial education.[5]

Further, students will have to apply their financial knowledge in a complex and rapidly changing financial environment. By the time they reach adulthood, the financial products available and related terminology will likely be radically different from when they were in school. The decisions they face may involve difficult trade-offs with strong emotional components as well as potentially misleading marketing from financial providers. Being able to critically evaluate financial options in such scenarios based on learning in school will require an especially deep and robust level of conceptual understanding, and knowledge of financial vocabulary will not be sufficient.

Thus, a high-quality education in finance should focus on developing conceptual understanding. Consistent with academic financial theory, the central concept in personal finance is the financial life cycle.[6] Therefore, financial education should include instruction in:

- How and why individuals and households transfer consumption over time.
- What financial risks individuals and households face and how they manage those risks.

These concepts are reflected in four essential understandings:

F1. The fundamental measure of financial wellbeing is wealth, or net worth. Financial statements are used to measure and track wealth through an analysis of assets, liabilities, income, and expense over the financial life cycle.

F2. The transfer of consumption forward and backward over time is generally financed via payment series, often with compound interest. Present value and future value are the tools used to evaluate payment series.

F3. Risk can be measured using probability and expected utility. These tools provide the means to evaluate risk management tools such as insurance.

F4. Investments in the stock market can be modeled using probability distributions to assess risk and return. These tools can be used to demonstrate that for sufficiently diversified stock market portfolios, over long investment horizons, the additional expected return increases more rapidly than the risk, increasing the probability of outperforming lower risk, lower-return investments.

Each of these financial understandings is directly related to a corresponding mathematical understanding, and the two should be used in concert with one another. The standards are designed so that the course can be viewed either from the perspective of a financial education course that utilizes mathematics or a mathematics course that utilizes applications to finance.

Each of the mathematical understandings has a *modeling component* – the process by which real-world phenomena are represented by mathematical relationships. While mathematical modeling is often viewed as an independent standard in high school mathematics, and often a neglected one, in this course modeling is integrated into all of these standards:

M1. The dynamics of financial transactions and wealth can be modeled using financial statements, which are mathematical models that utilize concepts of algebraic manipulation and linear relationships to capture the relationships between income, expense, assets, and liabilities.

M2. Time can be modeled mathematically using natural numbers, integers, rational numbers, and real numbers. The mathematics of time involves series, sequences, limits, exponents, logarithms, and other functional forms. In particular, the mathematics of compound interest involves exponential functions, their inverses, and geometric series.[7]

M3. The mathematics of financial risk can be modeled with random variables. Random variables represent a combination of probability and outcomes, often evaluated using expected value and other measures.

M4. Investment outcomes can be modeled with probability distributions, such as the binomial distribution and the normal distribution. The movement of stock prices is often modeled as a sequence of random variables sometimes called "random walks."[8] Generally, compound returns increase exponentially over time, while the standard deviation of such processes increases proportionally to the square root of time.

3. Making Mathematics Relevant

Math achievement levels in the US are lower than would be hoped, especially at the high school level. US students consistently rank poorly compared to those in other developed countries.[9]

Further, research demonstrates a strong link between mathematics achievements and future outcomes. In particular, high levels of mathematics education are associated with high income levels.[10]

A significant problem is student disengagement. In math class in particular, it's often reported that students are uninterested in the materials, and view math as unrelated to 'real life'. It has been shown, however, that learning is much more effective when students are highly engaged. One way to improve engagement is through the material being taught. Studies show improved results when students find the material relevant and interesting.[11]

Mathematics education leaves much room for improvement in this regard. Many so-called 'real-world' applications of math are contrived word problems, where the supposed application is irrelevant to the solution of the problem. What's needed are authentic applications, where the mathematical processes can be used to guide real life decision-making.

We believe that applications to finance are well-suited for this purpose. High school level mathematics is required to understand a range of financial problems that students will likely have to deal with in the future, making for an authentic application. In addition, students are interested in finance so they will find this work interesting and relevant.[12]

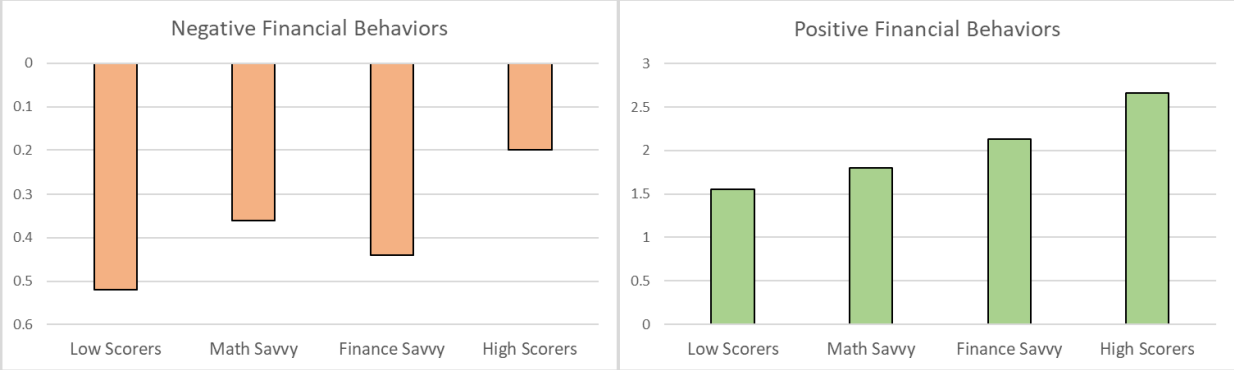
Moreover, many of the topics covered in K-12 mathematics were first developed or inspired by financial problems. This includes negative numbers, first introduced to represent a debt; concepts of equality featured in financial statements; limits associated with continuous compounding; and probability and statistics related to annuities.[13] Teaching these mathematical concepts in the context of finance returns them to their original development.

Integrating financial applications into a mathematics course will improve mathematics education by increasing engagement. The connection between mathematics and finance education is bi-directional, as a financial context helps students grasp and retain principles of mathematics, and mathematics skills enhance financial decision-making.

4. Financial Decision Making

Research we conducted with the FINRA Foundation and NORC at the University of Chicago indicates that people with both math skills and financial skills significantly outperform those with financial knowledge but not math skills.[14] People with both kinds of knowledge engage in significantly more positive financial behaviors and significantly fewer negative ones than people with financial knowledge alone.

Figure 1: Association Between Knowledge and Total Financial Actions Taken[15]



Financial education research more generally shows that people with financial education have greater financial confidence than those without and therefore engage in more financial activities across the board. Unfortunately, these people engage in more harmful financial activities as well as beneficial ones. Math knowledge is a differentiator, whereby negative actions are reduced and positive actions are retained and even increased. This aligns with our belief that math skills are required to build conceptual understanding in finance. Understanding the underlying mathematics allows people to critically analyze which of the myriad financial products offered will be beneficial to them.

5. Opportunity Cost

Our fifth principle states: "Student and teacher time is a scarce and valuable resource." There are only so many hours in the school year, both for students to learn, and for teachers to teach. And these hours are all, already, in use. Therefore, adding something to the curriculum requires removing something else. To justify such an addition, it must be shown that the benefits of adding the new course outweigh the costs of removing the old one.

This is a significant hurdle for implementing a standalone non-math-based course in personal finance. To do this, a course in an unrelated subject matter must be eliminated – from the arts, sciences, or humanities. For example, a financial education initiative in one US state required cutting the high school American history coursework in half.[16] Arguing that the benefits here clearly outweigh the costs is a tough case to make.

We would also caution against the "no cost option" of making finance education an extra-curricular course. Though this appears to offer the benefits of financial education without any cost, since nothing has to be cut, the appearance is deceptive. First, students taking one extra-curricular activity are potentially prevented from engaging in another – and there are many benefits to students participating in sports, creative arts, or other extra-curricular activities.

Second, an extra-curricular course is likely to be shorter and less comprehensive, which research shows is less effective.[17]

A math-based financial education course, on the other hand, does not require eliminating a course, but instead changing the focus of one high school mathematics course - for example a traditional Algebra 2 course or a fourth-year mathematics elective. The opportunity costs here concern any learning opportunities that are present in such a math course but are not in a financial mathematics course. And there certainly will be differences. For example, the sequence of mathematical topics in the standards presented below differs from a traditional algebra course in that these standards have less of a focus on polynomials and more of a focus on modeling and probability. In addition, it should be noted that a course in financial mathematics will probably cover fewer mathematical topics in total than a traditional one, since time must also be set aside to discuss the financial concepts and applications.

We do not believe these costs are severe, based on the previous discussion of best practices in mathematics education. Beyond mastering arithmetic, the significance of mathematics education lies primarily in developing mathematical *reasoning*, rather than acquainting students with as many mathematical concepts as possible: quality not quantity is the goal. Further, as spending time exploring mathematical applications to finance increases students' engagement with and understanding of mathematical concepts, the tradeoff is justified.

The mathematics topics covered by our standards are also particularly well suited to prepare students for college level mathematics. A firm understanding of sequences and series, the idea of manipulating equations as a matter of identity rather than 'solving for x,' and interpreting the input of a function as time are all crucial preparation for calculus. A fundamental understanding of random variables and probability distributions is an essential foundation for the math required in social science. Further, a combination of these elements is needed for stochastic calculus, which is required for a range of career paths in science, finance, and tech.

6. Unpacking the Principles

As we discussed in section 2, we believe that a high-quality personal financial education should provide an understanding of the financial and mathematical concepts necessary to navigate the financial life cycle.

For a person to achieve good financial outcomes, they must have the financial means to meet their goals across their whole lifetime. One's financial needs and abilities vary over the course of one's lifetime, and one must plan for this when making financial decisions; this is what we mean by 'navigating the financial life cycle'. To do this requires an understanding of:

How and why individuals and households transfer consumption over time.

- One's income varies across one's lifetime, as do one's necessary expenses, but they do not always line up, so one must use financial products to make sure one has the necessary resources available at different points in time to meet these needs.

What financial risks individuals and households face and how they manage those risks.

- One's future, financially speaking, is uncertain, and some possible outcomes would be undesirable if sufficient financial resources were unavailable. Financial products can be used to reduce the risk that the necessary resources will not be available.

As we discussed, fully grasping these ideas requires four essential financial understandings and the associated mathematical understandings.

F1. The fundamental measure of financial wellbeing is wealth, or net worth. Financial statements are used to measure and track wealth through an analysis of assets, liabilities, income, and expense over the financial life cycle.

- To think about navigating the financial life cycle, we need to know how to adequately measure a financial situation. This requires thinking in terms of wealth, which is computed as assets less liabilities, rather than only cash which is often the primary focus in discussions of budgeting. Students must understand that their wealth is the means by which they can meet their needs across the life cycle.
- Wealth is not just a concept for those who are financially well off, nor does wealth refer only to investments at financial institutions. Rather the concept of wealth applies to all of our decisions regarding spending, investing and borrowing and how we transform income today into consumption in the past and the future.

F2. The transfer of consumption forward and backward over time is generally financed via payment series, often with compound interest. Present value and future value are the tools used to evaluate payment series.

- Borrowing and investing involve compound interest, which makes understanding how to transfer consumption inseparable from a mastery of the mathematics of exponential functions and geometric series. Further, in real-life instances of transferring consumption, payments frequently will be made at regular intervals, rather than in lump sum. That means students must move beyond the simple calculations of compound interest that are typically taught, and work with interest within geometric series.

F3. Risk can be measured using probability and expected utility. These tools provide the means to evaluate risk management tools such as insurance.

- The fundamental concept when making any kind of decision under uncertainty is expected utility – and this includes financial decisions. To understand this, students must master the mathematics of probability and expected value. Combining the mathematical tools with a critical analysis of uncertain financial situations allows them

to properly understand the importance of insurance.

F4: Investments in the stock market can be modeled using probability distributions to assess risk and return. These tools can be used to demonstrate that for sufficiently diversified stock market portfolios, over long investment horizons, the additional expected return increases more rapidly than the risk, increasing the probability of outperforming lower risk, lower- return investments.

- Building significant financial wealth in the long term generally requires investing in the stock market, so it is essential for students to understand how to approach this in an informed manner.

We refer to our principles as "essential understandings" and begin many of our standards with the phrase "students understand". This is intended, on the mathematical side, to mean that students can do more than merely use a formula correctly or perform an operation or procedure with fluency; similarly, they can do more than replicate the definition of a financial concept or follow a prescriptive rule of thumb for budgeting. It means they know when to call upon this knowledge in non-routine situations, why one might need this knowledge, and they can explain or derive the underlying mathematics. It means students know why things work. In the context of financial math it also means students can connect the mathematics to the real world financial applications.

If a student *understands* a formula, then they know when to use the formula, why one might need the formula and they can derive the formula or make sense of the reasons the formula is composed as it is and how it relates to the real world situation it models. Students should be able to apply the formula in unfamiliar settings and recognize it when it is presented in a new form, for example if variables are represented by Greek rather than Roman letters. Similarly, if a student understands a financial concept, they should be able to think critically about how it factors into a range of situations, including with regard to new financial products they have not previously encountered.

Within the standards, we also provide key examples of how students can "use" their understanding. These detail key concrete financial skills students need and provide a demonstration of understanding. The *uses* we describe are not exhaustive as understanding is essentially open-ended. This makes it a high bar to clear, but also what's needed to navigate an open ended and constantly evolving financial world.

7. The Standards

The FiCycle Standards include both finance and mathematics standards. The Standards are built around four pairs of essential understandings (F1 & M1, F2 & M2, F3 & M3, and F4 & M4), with sub sections unpacking their content. Each pair of essential understanding is mutually reinforcing: the math content of M1 is required to understand the financial content of F1, while the financial content provides an authentic application of the math content. While the math and finance subsections are also interrelated, there is not a one-to-one connection between each math and finance subsection. (For example, F1.3 is not directly related to M1.3.)

The Plus (+) Standards offer additional mathematics that students should learn if their course pathway leads to advanced courses such as calculus, advanced statistics, or discrete mathematics.

<table>
<tr><td colspan="2">Finance F1. The fundamental measure of financial wellbeing is wealth, or net worth. Financial statements are used to measure and track wealth through an analysis of assets, liabilities, income, and expense over the financial life cycle.

Mathematics M1. The dynamics of financial transactions and wealth can be modeled using financial statements which are mathematical models that utilize concepts of algebraic manipulation and linear relationships to capture the relationships between income, expense, assets, and liabilities.</td></tr>
<tr><td>F1.1 Students know that wealth is defined as ability for consumption and understand its significance over and above cash balance.

F1.2 Students understand the four key concepts for measuring wealth: income, expense, assets, and liability. Students can use these concepts to classify transactions.

a. Students can track and calculate these values using financial statements: income statements, balance sheets, and budgeting tables.</td><td>M1.1 Students are able to manipulate and substitute into linear equations, including key financial applications:

a. The net worth equation:
$NW = A - L$
(net worth = assets – liabilities)
b. The net income equation:
$NI = I - E$
(net income = gross income – expenses)

M1.2 Students understand the relationship between an equation and a function taking elements of the equation as inputs. They apply this when relating financial equations to financial statements.</td></tr>
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<table>
<tr><td>F1.3 Students understand the key factors that influence wealth:
a. The connections between income level, career, and education.
b. The difference between incurring an expense and purchasing an asset.
c. The importance of maintaining a cash balance to preserve liquidity for emergencies.

The relationship between taxes and income.</td><td>For example, Students can create a function that gives net worth as an output using the asset and liabilities entries on a balance sheet as inputs.

M1.3 Students are comfortable dealing with units and percentages, in situations involving currency and taxes respectively.

M1.4 (+) Students understand piecewise functions and use them to model total tax, marginal tax rate, and effective tax rate.</td></tr>
<tr><td colspan="2">Finance F2. The transfer of consumption forward and backward over time is generally financed via payment series, often with compound interest. Present value and future value are the tools used to evaluate payment series.

Mathematics M2. Time can be modeled mathematically using natural numbers, integers, rational numbers, and real numbers. The mathematics of time involves series, sequences, limits, exponents, logarithms, and other functional forms. In particular, the mathematics of compound interest involves exponential functions, their inverses, and geometric series.</td></tr>
<tr><td>F2.1 Students understand the role of borrowing and investing in transferring consumption across the financial life cycle.
a. If one has a surplus of income now, one can invest it in order to use it at a future time when one’s consumption needs are higher than one’s income.
b. If one’s current expenses exceed one’s current income, one can borrow money to meet them and repay the money with future surplus income.</td><td>M2.1 Students understand the rules of exponents, including negative exponents, and are comfortable manipulating them in algebraic expressions.
a. Understand and use the compound interest formula:
$$PV = FV\left(1+\frac{r}{n}\right)^{n\bullet t}$$
b. Understand and use the discounting formula:
$$PV = FV\left(1+\frac{r}{n}\right)^{-n\bullet t}$$</td></tr>
</table>

F2.2 Students understand that generally the value of money changes over time due to interest: a dollar today is worth more than a dollar in the future.

a. Interest is earned or paid as a percentage of the value being transferred.
b. The connection between present value and future value is calculated using the compound interest equation.

F2.3 Students know the different borrowing needs one may face and can analyze financial instruments for meeting those needs.

a. Buying a house builds wealth, through spending money on housing equity, an asset, rather than rent, an expense.
b. A mortgage is a loan used for buying a house.
c. Spending money on college may lead to a future career with a higher income.
d. The costs of education can be covered using student loans.
e. A credit card can be used to borrow small amounts of money instantly but generally has a higher interest rate than other loans offered by financial institutions.
f. Credit scores influence whether one can access a variety of borrowing opportunities and the favorability of the terms offered; credit scores are determined by past financial behavior.

M2.2 Students understand Euler's number, and use it in the continuous compounding formula:

$$FV = PV \bullet e^{rt}$$

a. (+) Students understand the definition of Euler's number as a limit and use this to derive the continuous compounding formula.

M2.3 Students understand the importance of estimation and how to employ it effectively. Students apply this to using the rule of 72 to estimate the time it takes for an investment of debt to double in value:

$$t = \frac{72}{R \bullet 100}$$

a. Students understand the rules for logarithms, including natural logs, and use this to understand why the rule of 72 works as an approximation.
b. Students can estimate one of: the time, rate, or multiple of the initial investment given the other two using the rule of 72.

F2.4 Students know the different needs for investing one may have and the financial instruments for meeting those needs.

a. Savings accounts allow one to invest money with very low risk, high flexibility, and low interest rates.
b. Treasury Bills, Notes and Bonds are tools for investing at various maturities, with near zero risk of loss of principal, and modest interest rates.
c. Stocks are higher risk investments with higher expected returns on average.
d. Getting a mortgage usually requires a down payment, which one will often have to save money for.
e. Upon retiring, one will no longer have wage income to meet one's living expenses, so one will often prepare for this by investing in a retirement account while working.
f. One may have to deal with unexpected expenses at any point in one's life cycle; one can prepare for these savings by having an easily accessible emergency fund.

F2.5 Students know that investments and repayments typically occur through a series of payments over an extended period of time and understand how this affects financial decisions. They can describe situations involving the following features:

a. Annuities and amortization
b. Growing Payment series
c. Inflation

M2.4 Students understand how to calculate and model with arithmetic and geometric series and can apply them to payment series appropriately.

a. Series of simple interest payments can be modeled with an arithmetic series.
b. Series of compound interest payments can be modeled with a geometric series.
c. Series can be created using recursive and explicit formulas for sequences.
d. When modeling realistic examples, one must add additional variables for growth rate and inflation.

M2.5 Students understand how to break down complex formulas into simpler constituents. They can apply this to complex payment series formulas.

Finance F3. *Risk can be measured using probability and expected utility. These tools provide the means to evaluate risk management tools such as insurance.*

Mathematics M3. *The mathematics of financial risk can be modeled with random variables. Random variables represent a combination of probability and outcomes and are often evaluated using expected value and other measures.*

F3.1 Students understand the different kinds of financial risk one faces over the financial life cycle:

a. Healthcare costs
b. Vehicle damage
c. Property damage/theft
d. Device damage/malfunction
e. Dependent impoverishment due to death of caregiver

F3.2 Students understand how insurance mitigates risk, and how the nature of the risk affects the need to purchase insurance.

a. Students understand that insurance involves one party assuming another's risk. Students can distinguish between diversified vs correlated risk.
b. Students understand and can identify catastrophic risk.

F3.3 Students can describe different kinds of insurance and their distinguishing features.

a. Health insurance
b. Car insurance
c. Home insurance
d. Device insurance
e. Life insurance

M3.1 Students understand the fundamental features of probability and use this to measure financial risk.

a. $P(Sample\ Space) = 1$
b. $P(E) = \frac{\#(Outcomes\ in\ which\ E\ occurs)}{\#(Outcomes\ in\ the\ experiment)}$
c. If a and b are independent events, then $P(a\&b) = P(a) \cdot P(b)$

M3.2 Students understand the concepts of expected value and expected utility and apply them to financial decisions:

a. Expected Value Formula:
$$EV = \sum_{i=1}^{n} P(o_i) \bullet V(o_i)$$
b. Expected Utility Formula:
$$EU = \sum_{i=1}^{n} P(o_i) \bullet U(o_i)$$

M3.3 Students understand binomial distributions and calculate outcomes' probability in binomial experiments. They are able to use this to model situations with diversified risk.

a. Binomial experiments can be represented by binomial trees.
b. Binomial Theorem:

$$P(k\ successes) = \binom{n}{k} p^k (1-p)^{n-k}$$

Finance F4. *Investments generally and specifically in the stock market involve a trade-off between risk and return. For sufficiently diversified stock market portfolios, over long investment horizons, the additional expected return increases more rapidly than the risk, increasing the probability of outperforming lower risk, lower-return investments. However, long investment horizons do not eliminate the risk of stock market investments.*

Mathematics M4. *Investment outcomes can be modeled with probability distributions, such as the binomial distribution and the normal distribution. The movement of stock prices is often modeled as a sequence of random variables sometimes called "random walks." Generally, compound returns increase exponentially over time, while the standard deviation of such processes increases proportionally to the square root of time.*

F4.1 Students understand the nature of investments in the stock market, and the specific concepts of shares, dividends, and returns.

F4.2 Students understand the Efficient Market Hypothesis, and the implications it has for investment decisions, in particular, how rapidly market prices adjust to new information and thereby limit opportunities to earn excess profits using public information.

F4.3 Students understand the relationship between risk and return: that investors are generally risk averse, and so higher risk investments generally offer higher expected returns.

F4.4 Students relate their knowledge of stocks and risk to their understanding of the financial life cycle to evaluate when investing in stocks is appropriate and when lower risk strategies are appropriate.

M4.1 Students understand that stock price changes can be modeled as random walks and that random walks can be represented as binomial trees and evaluated using the binomial distribution.

M4.2 Students understand the concepts of mean and standard deviation and can relate this to financial risk.

M4.3 Students understand the key features of the normal distribution, and the situations in which the binomial distribution approximates the normal distribution.

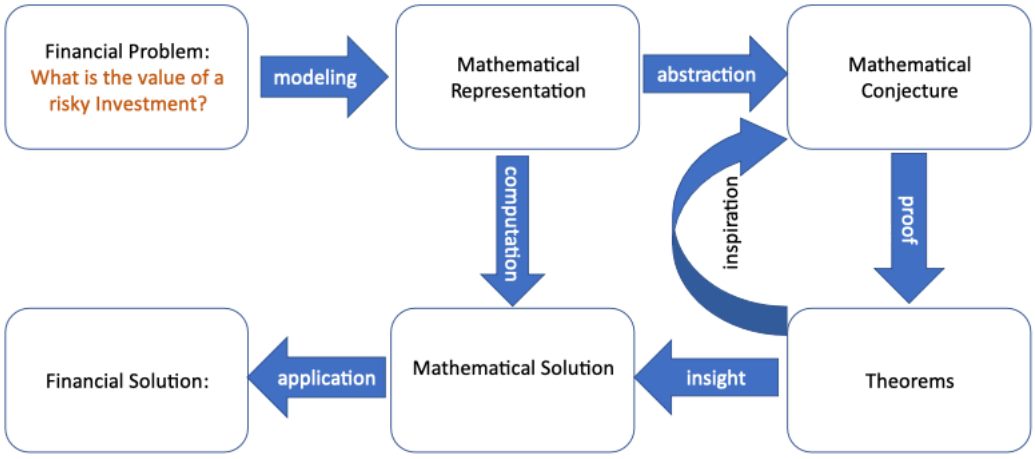
M4.4 Students can mathematically model investments with different portfolio sizes over different lengths of time using the binomial distribution and observe general patterns between these factors and financial risk.

8. A Note on Modeling

While we believe pedagogy and best practices in education are of vital importance, this document does not address these issues. These are content standards that describe the concepts we believe need to be taught to deliver a high-quality education in personal finance and mathematics, and they should be combined with an effective pedagogical strategy, such as those in the Common Core "Standards for Math Practice." For further guidance in this area, we should look to the current best practices in the mathematics education community.

That said, the central role modeling plays in our standards and has strong connections to pedagogy. Each of our standards is rooted in financial challenges, alongside the relevant mathematical models of those challenges and the mathematics needed to evaluate the financial challenge in the context of the model. For example, modeling investments utilizing the concepts of random walks and efficient markets leads to the mathematics of martingales which we explore via binomial trees. The study of these representations has led to significant advances in mathematics both within and outside of finance.

The schematic below shows our thought process.[18]



We believe that creating mathematical representations through modeling serves not just to create solutions to real world problems, but also to explore mathematical ideas in more depth. Financial problems are an ideal context for employing student exploration and discovery as a pedagogical strategy, so that students learn to construct models for themselves rather than replicating a model they have been presented with previously. The authentic nature of the problems allows students to critically reason about what mathematical tools to apply, and become more engaged in learning the mathematical abstractions associated with that representation. In this way a course in personal finance provides a great opportunity to

build sophisticated modeling skills and mathematical understanding.

9. Equity in Personal Finance and Mathematics Education

The need for equitable standards and curricula is well known in the education sphere, and mathematics education in particular.

Students come into the classroom with different backgrounds and may respond differently to the materials. Equitable education should be designed to be responsive to these different backgrounds, so all students have an equal opportunity to learn and be engaged.

These considerations are particularly important when it comes to financial education, where issues of equity are particularly salient, given current and historical financial inequality and injustice in the US. Key factors to consider are:

- Equitable language use: Do not use potentially derogatory language related to various financial outcomes.
- Equitable background knowledge: Make sure the background knowledge and vocabulary assumed in materials fits with the student body, and is not specific to, e.g., white middle-class students.
- Judgment free: Do not present scenarios in a judgmental manner, suggesting that people are to blame for negative financial outcomes. Acknowledge that many people face very difficult financial decisions, and "smart budgeting" does not guarantee prosperity.
- Historical acknowledgment: Current distributions of wealth are strongly influenced by past injustice, and a young person's future financial opportunities are significantly impacted by their family's wealth.

If students feel that they or their family or community members are being judged or belittled for their financial situation, they may be unwilling to engage with the course or feel that it is not relevant to them. If there is not an authentic acknowledgment of the inequities around them it may not feel relevant to their actual lives, and instead be just another artificial 'real world problem' they have to learn how to answer test questions about.

Creating an equitable course in personal finance is, therefore, highly context dependent as it requires responding to the needs of the specific students being taught. It's not something that can be addressed through a one-size-fits-all set of standards. Instead, it's something each educator must pay attention to in the classroom.

10. Concluding Remarks

In our work, we've met and been influenced by countless educators working in varied settings with students from all walks of life and backgrounds. Their efforts have both been an

inspiration to us and informed our work. We are deeply grateful for all the dedicated educators and professionals working on these topics in education.

We recognize that there are many challenges required to implement high-quality financial education across the US: from providing educators and students with the training and resources they need, to understanding how to tailor materials to the specific and varied backgrounds of students across the country – especially responding to the experiences of historically marginalized groups. We are excited to collaborate and learn as we address these questions, and we firmly believe that the conceptually grounded approach outlined in our standards is an essential foundation for this project.

Our goal is to change students' experience of personal finance education, and our standards are the starting point, not the end goal. It is our sincerest hope that this document can serve as a tool for educators, schools, and other organizations seeking to create, evaluate, or adopt mathematical and financial education materials. We hope to support and learn from those who take up this challenge.

Notes:

[1] See, Hasler, A., Lusardi, A., & Oggero, N. (2018). Financial fragility in the US: Evidence and implications. Global Financial Literacy Excellence Center, The George Washington University School of Business. https://gflec.org/wp-content/ uploads/2018/04/Financial-Fragility-Research- Paper-04-16-2018-Final.pdf

[2] See Lusardi, A., & Mitchell, O. S. (2014). The economic importance of financial literacy: Theory and evidence. Journal of Economic Literature, 52(1), 5-44. https://doi.org/10.1257/jel.52.1.5

[3] We discuss this further in our academic literature review FiCycle 2019 "Combining Mathematics and Financial Education". (https://ficycle.org/wp- content/uploads/2022/02/FiCycle-Research-Overview-2022.pdf)

[4] See, Bullmaster Day (2006) A Synthesis of Research on Effective Mathematics Instruction. Touro College Graduate School of Education.
https://www.academia.edu/7818450/A_Synthesis_of_Research_on_Effective_Mathemati cs_Instruction

[5] We discuss the importance of conceptual understanding in financial education in depth in our working paper FiCycle (2018) "Financial Education and Conceptual Understanding: Learning from Best Practices in Mathematics" https://ficycle.org/wp-content/uploads/2021/12/conceptual- understanding.pdf

[6] Ando, A., & Modigliani, F. (1963). The" life cycle" hypothesis of saving: Aggregate implications and tests. The American economic review, 53(1), 55-84.

[7] The mathematics of continuously compounded interest also has an analogue in the exponential representation of trigonometric functions.

[8] Certain sequences of random variables are called martingales and are a central theme of advanced mathematical finance.

[9] See PISA 2018 Results https://www.oecd.org/pisa/Combined_Executive_Summaries_PISA_2018.pdf

[10] Goodman, J. (2019). The labor of division: Returns to compulsory high school math coursework. Journal of Labor Economics, 37(4), 1141-1182

[11] See, Greene, B. A., Miller, R. B., Crowson, H. M., Duke, B. L., & Akey, K. L. (2004). Predicting high school students' cognitive engagement and achievement: Contributions of classroom perceptions and motivation. Contemporary educational psychology, 29(4), 462-482.

[12] We discuss how mathematics engagement can be enhanced by financial applications in depth in our working paper "Enriching Mathematics through an Application to Finance" https://ficycle.org/wp-content/uploads/2021/12/Enriching-Math.pdf

[13] See momath.org/buck for a series of discussions of these connections.

[14] "They Just Add Up: Combined Math and Financial Knowledge Tied to Better Financial Outcomes" https://www.finrafoundation.org/sites/finrafoundation/files/Combined-Math-and- Financial-Knowledge-Tied-to-Better-Financial-Outcomes.pdf

[15] Chart taken from https://www.finrafoundation.org/sites/finrafoundation/files/Combined- Math-and-Financial-Knowledge-Tied-to-Better-Financial-Outcomes.pdf

[16] See https://www.wfae.org/local-news/2019-07-08/governor-signs-bill-making-finance-course- a-graduation-requirement

[17] See, Urban, C., Schmeiser, M., Collins, J. M., & Brown, A. (2020). The effects of high school personal financial education policies on financial behavior. *Economics of Education Review*, *78*, 101786.

[18] This diagram comes from a series we did with the Museum of Math (momath.org/buck)