FiCycle Standards for Personal Finance and Mathematics

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1. Introduction to the Standards

At Financial Life Cycle Education (FiCycle), we believe it is essential for students to receive a high-quality education in both personal finance and mathematics. Further, we believe that offering students a mathematics course that is based upon applications to finance advances both goals. In this document, we present a set of standards for a high school course in personal finance and the associated high school level mathematics. Our approach is responsive to a greater focus in the math education community on rigorous alternative pathways for high school math education. In particular, the call for standards for financial mathematics that link the two subjects conceptually rather than independent mathematics and personal finance standards.

The Standards focus on the concept of wealth as a measure of potential for consumption, how consumption can be transferred across time through investing and borrowing, and on measuring and managing financial risk through the use of insurance and stock market investing. The Standards address both the financial and mathematical dimensions of these concepts. Wealth is also referred to as net worth and includes financial assets plus real assets minus liabilities. Wealth is a useful perspective for evaluating financial transactions regardless of an individual's level of income or financial capacity. (See Section 4: Equity in Personal Finance and Mathematics Education.)

By teaching students the financial and mathematical concepts to understand these fundamental ideas, they will be equipped to rationally analyze financial decisions they face in life, rather than relying on rules of thumb. They will also learn how mathematics can be authentically and rigorously applied to real life concerns.

2. The Standards

The FiCycle Standards include both finance and mathematics standards. The Standards are built around four pairs of essential understandings (F1 & M1, F2 & M2, F3 & M3, and F4 & M4), with sub sections unpacking their content. Each pair of essential understanding is mutually reinforcing: the math content of M1 is required to understand the financial content of F1, while the financial content provides an authentic application of the math content. While the math and finance subsections are also interrelated, there is not a one-to-one connection between each math and finance subsection. (For example, F1.3 is not directly related to M1.3.)

<table>
<tr><td colspan="2">Finance F1. The fundamental measure of financial wellbeing is wealth, or net worth. Financial statements are used to measure and track wealth through an analysis of assets, liabilities, income, and expense over the financial life cycle.

Mathematics M1. The dynamics of financial transactions and wealth can be modeled using financial statements which are mathematical models that utilize concepts of algebraic manipulation and linear relationships to capture the relationships between income, expense, assets, and liabilities.</td></tr>
<tr><td>F1.1 Students know that wealth is defined as ability for consumption and understand its significance over and above cash balance.

F1.2 Students understand the four key concepts for measuring wealth: income, expense, assets, and liability. Students can use these concepts to classify transactions.

a. Students can track and calculate these values using financial statements: income statements, balance sheets, and budgeting tables.</td><td>M1.1 Students are able to manipulate and substitute into linear equations, including key financial applications:

a. The net worth equation:
$NW = A - L$
(net worth = assets – liabilities)

b. The net income equation:
$NI = I - E$
(net income = gross income – expenses)

M1.2 Students understand the relationship between an equation and a function taking elements of the equation as inputs. They apply this when relating financial equations to financial statements. For example, Students can create a function that gives net worth as an output using the asset and liabilities entries on a balance sheet as inputs.</td></tr>
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F1.3 Students understand the key factors that influence wealth: a. The connections between income level, career, and education. b. The difference between incurring an expense and purchasing an asset. c. The importance of maintaining a cash balance to preserve liquidity for emergencies. d. The relationship between taxes and income.	M1.3 Students are comfortable dealing with units and percentages, in situations involving currency and taxes respectively. M1.4 (+) Students understand piecewise functions and use them to model total tax, marginal tax rate, and effective tax rate.
Finance F2. *The transfer of consumption forward and backward over time is generally financed via payment series, often with compound interest. Present value and future value are the tools used to evaluate payment series.* **Mathematics M2.** *Time can be modeled mathematically using natural numbers, integers, rational numbers, and real numbers. The mathematics of time involves series, sequences, limits, exponents, logarithms, and other functional forms. In particular, the mathematics of compound interest involves exponential functions, their inverses, and geometric series.*	
F2.1 Students understand the role of borrowing and investing in transferring consumption across the financial life cycle. a. If one has a surplus of income now, one can invest it in order to use it at a future time when one's consumption needs are higher than one's income. b. If one's current expenses exceed one's current income, one can borrow money to meet them and repay the money with future surplus income.	M2.1 Students understand the rules of exponents, including negative exponents, and are comfortable manipulating them in algebraic expressions. a. Understand and use the compound interest formula: $FV = PV\left(1+\frac{r}{n}\right)^{n \bullet t}$ b. Understand and use the discounting formula: $PV = FV\left(1+\frac{r}{n}\right)^{-n \bullet t}$

F2.2 Students understand that generally the value of money changes over time due to interest: a dollar today is worth more than a dollar in the future.

a. Interest is earned or paid as a percentage of the value being transferred.
b. The connection between present value and future value is calculated using the compound interest equation.

F2.3 Students know the different borrowing needs one may face and can analyze financial instruments for meeting those needs.

a. Buying a house builds wealth, through spending money on housing equity, an asset, rather than rent, an expense.
b. A mortgage is a loan used for buying a house.
c. Spending money on college gives one a qualification that can lead to a future career with a higher income.
d. The costs of education can be covered using student loans.
e. A credit card can be used to borrow small amounts of money instantly but generally has a higher interest rate than other loans offered by financial institutions.
f. Credit scores influence whether one can access to a variety of borrowing opportunities and the favorability of the terms offered; credit scores are determined by past financial behavior.

M2.2 Students understand Euler's number, and use it in the continuous compounding formula:

$$FV = PV \bullet e^{rt}$$

a. (+) Students understand the definition of Euler's number as a limit and use this to derive the continuous compounding formula.

M2.3 Students understand the importance of estimation and how to employ it effectively. Students apply this to using the rule of 72 to estimate the time it takes for an investment of debt to double in value:

$$t = \frac{72}{R \bullet 100}$$

a. Students understand the rules for logarithms, including natural logs, and use this to understand why the rule of 72 works as an approximation.
b. Students can estimate one of: the time, rate, or multiple of the initial investment given the other two using the rule of 72.

F2.4 Students know the different needs for investing one may have and the financial instruments for meeting those needs. a. Savings accounts allow one to invest money with very low risk, high flexibility, and low interest rates. b. Treasury Bills, Notes and Bonds are tools for investing at various maturities, with near zero risk of loss of principal, and modest interest rates. c. Stocks are higher risk investments with higher expected returns on average. d. Getting a mortgage usually requires a down payment, which one will often have to save money for. e. Upon retiring, one will no longer have wage income to meet one's living expenses, so one will often prepare for this by investing in a retirement account while Working. f. One may have to deal with unexpected expenses at any point in one's life cycle; one can prepare for these savings by having an easily accessible emergency fund. F2.5 Students know that investments and repayments typically occur through a series of payments over an extended period of time and understand how this affects financial decisions. They can describe situations involving the following features: a. Annuities and amortization b. Growing Payment series c. Inflation	M2.4 Students understand how to calculate and model with arithmetic and geometric series and can apply them to payment series appropriately. a. Series of simple interest payments can be modeled with an arithmetic series. b. Series of compound interest payments can be modeled with a geometric series. c. Series can be created using recursive and explicit formulas for sequences. d. When modeling realistic examples, one must add additional variables for growth rate and inflation. M2.5 Students understand how to break down complex formulas into simpler constituents. They can apply this to complex payment series formulas.

Finance F3. *Risk can be measured using probability and expected utility. These tools provide the means to evaluate risk management tools such as insurance.*

Mathematics M3. *The mathematics of financial risk can be modeled with random variables. Random variables represent a combination of probability and outcomes and are often evaluated using expected value and other measures.*

F3.1 Students understand the different kinds of financial risk one faces over the financial life cycle:

a. Healthcare costs
b. Vehicle damage
c. Property damage/theft
d. Device damage/malfunction
e. Dependent impoverishment due to death of caregiver

F3.2 Students understand how insurance mitigates risk, and how the nature of the risk affects the need to purchase insurance.

a. Students understand that insurance involves one party assuming another's risk. Students can distinguish between diversified vs correlated risk.
b. Students understand and can identify catastrophic risk.

F3.3 Students can describe different kinds of insurance and their distinguishing features.

a. Health insurance
b. Car insurance
c. Home insurance
d. Device insurance
e. Life insurance

M3.1 Students understand the fundamental features of probability and use this to measure financial risk.

a. $P(Sample\ Space) = 1$
b. $P(E) = \frac{\#(Outcomes\ in\ which\ E\ occurs)}{\#(Outcomes\ in\ the\ experiment)}$
c. If a and b are independent events, then $P(a\&b) = P(a) \cdot P(b)$

M3.2 Students understand the concepts of expected value and expected utility and apply them to financial decisions:

a. Expected Value Formula**:**

$$EV = \sum_{i=1}^{n} P(o_i) \bullet V(o_i)$$

b. Expected Utility Formula:

$$EU = \sum_{i=1}^{n} P(o_i) \bullet U(o_i)$$

M3.3 Students understand binomial distributions and calculate outcomes' probability in binomial experiments. They are able to use this to model situations with diversified risk.

a. Binomial experiments can be represented by binomial trees.
b. Binomial Theorem:

$$P(k\ successes) = \binom{n}{k} p^k (1-p)^{n-k}$$

Finance F4. *Investments generally and specifically in the stock market involve a trade-off between risk and return. For sufficiently diversified stock market portfolios, over long investment horizons, the additional expected return increases more rapidly than the risk, increasing the probability of outperforming lower risk, lower-return investments. However, long investment horizons do not eliminate the risk of stock market investments.*

Mathematics M4. *Investment outcomes can be modeled with probability distributions, such as the binomial distribution and the normal distribution. The movement of stock prices is often modeled as a sequence of random variables sometimes called "random walks." Generally, compound returns increase exponentially over time, while the standard deviation of such processes increases proportionally to the square root of time.*

F4.1 Students understand the nature of investments in the stock market, and the specific concepts of shares, dividends, and returns.	M4.1 Students understand that stock price changes can be modeled as random walks and that random walks can be represented as binomial trees and evaluated using the binomial distribution.
F4.2 Students understand the Efficient Market Hypothesis, and the implications it has for investment decisions, in particular, how rapidly market prices adjust to new information and thereby limit opportunities to earn excess profits using public information.	M4.2 Students understand the concepts of mean and standard deviation and can relate this to financial risk.
F4.3 Students understand the relationship between risk and return: that investors are generally risk averse, and so higher risk investments generally offer higher expected returns.	M4.3 Students understand the key features of the normal distribution, and the situations in which the binomial distribution approximates the normal distribution.
F4.4 Students relate their knowledge of stocks and risk to their understanding of the financial life cycle in order to evaluate when investing in stocks is appropriate and when lower risk strategies are appropriate.	M4.4 Students can mathematically model investments with different portfolio sizes over different lengths of time using the binomial distribution and observe general patterns between these factors and financial risk.

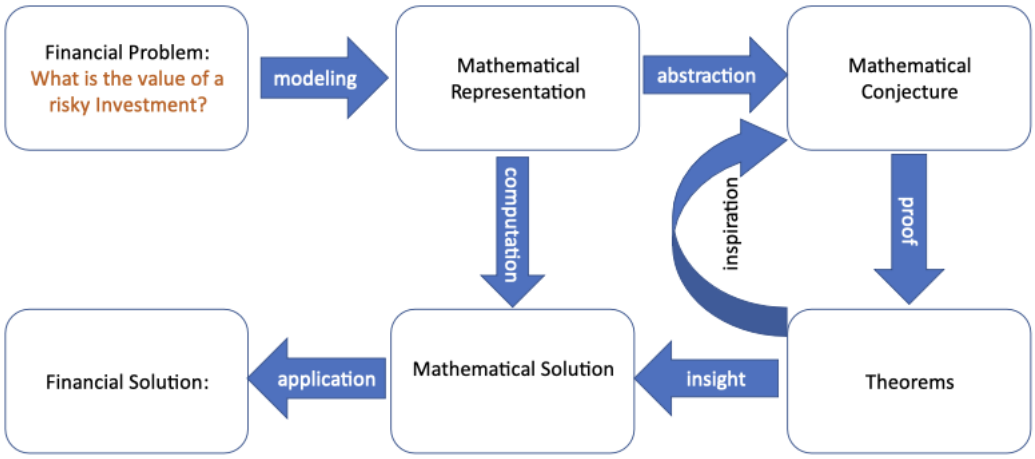
The Plus (+) Standards offer additional mathematics that students should learn if their course pathway leads to advanced courses such as calculus, advanced statistics, or discrete mathematics.

3. A Note on Modeling

While we believe pedagogy and best practices in education are of vital importance, this document does not address these issues. These are content standards that describe the concepts we believe need to be taught in order to deliver a high-quality education in personal finance and mathematics, and they should be combined with an effective pedagogical strategy, such as those in the Common Core "Standards for Math Practice." For further guidance in this area, we should look to the current best practices in the mathematics education community.

That said, the central role modeling plays in our standards and has strong connections to pedagogy. Each of our standards is rooted in financial challenges, alongside the relevant mathematical models of those challenges and the mathematics needed to evaluate the financial challenge in the context of the model. For example, modeling investments utilizing the concepts of random walks and efficient markets leads to the mathematics of martingales which we explore via binomial trees. The study of these representations has led to significant advances in mathematics both within and outside of finance.

The schematic below shows our thought process.[18]



We believe that creating mathematical representations through modeling serves not just to create solutions to real world problems, but also to explore mathematical ideas in more depth. Financial problems are an ideal context for employing student exploration and

discovery as a pedagogical strategy, so that students learn to construct models for themselves rather than replicating a model they have been presented with previously. The authentic nature of the problems allows students to critically reason about what mathematical tools to apply, and become more engaged in learning the mathematical abstractions associated with that representation. In this way a course in personal finance provides a great opportunity to build sophisticated modeling skills and mathematical understanding.

4. Equity in Personal Finance and Mathematics Education

The need for equitable standards and curricula is well known in the education sphere, and mathematics education in particular.

Students come into the classroom with different backgrounds and may respond differently to the materials. Equitable education should be designed to be responsive to these different backgrounds, so all students have an equal opportunity to learn and be engaged.

These considerations are particularly important when it comes to financial education, where issues of equity are particularly salient, given current and historical financial inequality and injustice in the US. Key factors to consider are:

- Equitable language use: Do not use potentially derogatory language related to various financial outcomes.
- Equitable background knowledge: Make sure the background knowledge and vocabulary assumed in materials fits with the student body, and is not specific to, e.g., white middle-class students.
- Judgment free: Do not present scenarios in a judgmental manner, suggesting that people are to blame for negative financial outcomes. Acknowledge that many people face very difficult financial decisions, and "smart budgeting" does not guarantee prosperity.
- Historical acknowledgment: Current distributions of wealth are strongly influenced by past injustice, and a young person's future financial opportunities are significantly impacted by their family's wealth.

If students feel that they or their family or community members are being judged or belittled for their financial situation, they may be unwilling to engage with the course or feel that it is not relevant to them. If there is not an authentic acknowledgment of the inequities around them it may not feel relevant to their actual lives, and instead be just another artificial 'real world problem' they have to learn how to answer test questions about.

Creating an equitable course in personal finance is, therefore, highly context dependent as it

requires responding to the needs of the specific students being taught. It's not something that can be addressed through a one-size-fits-all set of standards. Instead, it's something each educator must pay attention to in the classroom.

5. Conclusion

At FiCycle, our goal is to change students' experience of personal finance education, and these standards are the starting point, not the end goal. It is our sincerest hope that this document can serve as a tool for educators, schools, and other organizations seeking to create, evaluate, or adopt mathematical and financial education materials. We hope to support and learn from those who take up this challenge.