Spreadsheets as an Effective Use of Technology in Mathematics Education

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The use of technology is an essential component of high quality mathematics education. However, the use must be appropriate in order to be effective. Research suggests that spreadsheet software is equipped to be a particularly fruitful application of technology in the mathematics classroom. The use of spreadsheets allows students to deepen their understanding of both algebra and modeling. For this understanding to occur, though, the technology must be purposefully integrated into the mathematics curriculum to ensure students can appreciate its utility. This paper aims to increase our understanding of the role of spreadsheets in mathematics education and connect their use explicitly to research surrounding best practices. First, we present an argument for the value of spreadsheets in mathematics education, making explicit connections with research about best practices for the use of technology in mathematics. Second, we outline a strategy for incorporating spreadsheets within the high school mathematics classroom through a lesson series on retirement savings, in order to demonstrate how spreadsheets bring about a wide range of educational benefits.

1. Introduction

It is widely acknowledged that the use of technology is an important component of mathematics education, but also that the use must be *appropriate* in order to be effective (NCTM 2015). In addition, recent research suggests that spreadsheet software is equipped to be a particularly fruitful application of technology in the mathematics classroom. The use of spreadsheets allows students to deepen their understanding of both algebra and modeling. For this understanding to occur, though, the technology must be purposefully integrated into the mathematics curriculum to ensure students can appreciate its utility. Though much research has explored why and how spreadsheets can be used in mathematics lessons, there is a need to further articulate how to connect these findings with broader research on best practices in education.

This paper aims to increase our understanding of the role of spreadsheets in mathematics education and connect their use explicitly to research surrounding best practices. First we present a research informed theoretical argument for the value of spreadsheets in mathematics education. In doing this, we make connections with research on the effective use of technology in mathematics. Second, we outline a strategy for incorporating spreadsheets within the high school mathematics classroom through a lesson series on retirement savings. In doing this, we demonstrate how spreadsheets bring about a wider range of educational benefits. Finally, we review qualitative evidence on how such a lesson series is perceived by high school mathematics teachers.

2. Technology in Mathematics Education

NCTM's *Principles to Actions* (2014) emphasizes the importance of technology in mathematics education: one of their five essential elements of effective teaching is the "appropriate use of tools and technology." The fact that the use of technology must be *appropriate* is a crucial point:

> The value of the technology depends on whether students actually engage with specific technologies or tools in ways that promote mathematical reasoning and sense making… teachers may merely teach students procedures for using tools or technology to solve problems…Also, textbooks and curricular materials may claim to incorporate tools and technology but fail to do so in ways that help teachers promote reasoning and sense making. (NCTM 2014 pp. 80-81)

The key point we will emphasize and return to is that technology must be used to "promote mathematical reasoning and sense making," in order to be an effective tool in education. This requires the technology to be a tool for enhancing mathematical *reasoning*, not just computation. NCTM further emphasizes this point in its 'Statement on Technology' (2015), as does other influential research on the use of technology in mathematics education (Dick & Hollebrands

2011). The mathematics at work must be on display for the students to interact with, rather than occurring 'behind the scenes.' Using the technology must constitute doing mathematics, rather than being a mere gimmick, only tangentially related to the mathematical content of the lesson.

The uses of technology which fit these criteria are best illustrated through example. First, consider a teacher of geometry engaging students in a unit on 3D objects. Consider how in many video games, incredibly sophisticated mathematics is employed in simulating the interactions of objects in a physical environment. One could imagine a lesson in which students are instructed to play such a game in an attempt to teach mathematics through technology. However, the mathematics in question is not on display when playing the game; it is largely invisible to the user. As a result, playing the game does not, by itself, promote mathematical reasoning and sense making, and so such a lesson would not be an effective use of technology.

In contrast, consider a lesson that uses the popular online technology "Desmos" to explore the graphs of polynomial functions.[1] Students can enter a number of polynomial equations and the technology will instantly generate their graphs. They can explore how altering different components of the equation affects the graph, allowing them to understand the relationship between an equation and its graph in an interactive fashion. In this case, the technology does promote mathematical reasoning and sense making.

An ancillary benefit in this example is that it allows students to create a large number of graphs in a short span of time, so they can notice patterns more easily. They can avoid focusing on the procedural task of plotting points and sketching graphs, and as a result can dedicate more time to building conceptual understanding regarding the relationship between equations and graphs. This is often a feature of effective use of technology: it allows students to avoid spending most of their time on procedural tasks so they can develop conceptual understanding, in line with best practices in mathematics education (NCTM 2014).[2]

On the other hand, learning to use a technology sometimes requires developing procedural knowledge, and this creates an impediment to mathematical reasoning. For example, computer programming, while in the long run a valuable field for mathematical reasoning, has a very steep learning curve. Learning the language is a time and effort intensive process that must be undertaken before a student can 'get to the math.' For example, if one wants to create a

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[1] For an example of this kind of lesson can be found at the link see Desmos (2018).

[2] As an anonymous reviewer notes, for certain graphing activities, Excel can be a superior option to Desmos. For example, if one rotates a 2d object in Desmos, the program returns a static image which hides the mathematics of the transformation. However, a rotation matrix in Excel comes with a scroll bar, which allows the user to engage with the mathematical process interactively.

geometric series with 20 terms, first term 3 and common ratio 1.2 in the mathematical programming language 'R', one must input the following instructions in the console:

1. n<-1:20
2. a<-3
3. r<-1.2
4. geo_term<-function(n){a*r^n}
5. geo_seq<-sapply(n,geo_term)
6. geo_ser<-sum(geo_seq)
7. geo_ser

This requires, among other things, understanding how to create functions in R and using the 'sapply' function, which are likely unfamiliar to students and do not connect with high school mathematics in an intuitive manner. For this reason, if sufficient time is not allocated to the project, this kind of technology may not be an effective technology for mathematics education. (In section 4 we demonstrate how geometric sequences can be created in Excel in a much more intuitive manner.)

One particularly important place where there is a role for technology is *mathematical modeling*. The Common Core State Standards (CCSS) on high school modeling state that "Modeling links classroom mathematics and statistics to everyday life, work, and decision-making… When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data." (CCSS 2019)

To summarize, mathematics educators must find ways to effectively harness technology. The key issue here is that the technology must play a role in mathematical reasoning. In addition, particular attention must be paid to using technology when engaging in mathematical modeling as it is a particularly fruitful area to apply these principles and ideas.

3. Why Spreadsheets?

We will argue that spreadsheet software is a form of technology that fits the conditions for effective mathematics education. First we must provide some background on the technology itself. Spreadsheet software, most prominently Microsoft Excel, provides an interactive matrix of cells, used primarily to perform calculations on data – though both numbers and text can be input. Of particular significance is the fill function, which allows you to apply the same function to a series of cells, but transposes the cells referenced as you fill up, down or across. The workings and power of the fill function are discussed in Baker and Baker (2010). As we will explain, these features together make spreadsheets a technology well-suited to mathematics education.

A first point to note is that due to the intuitive display and input system, learning the language is not a significant barrier to using this technology to do mathematics. In virtue of this, spreadsheets seem a better technological aid for mathematical *reasoning* than programming activities. As Steward argues, "students find it easier and quicker to use a spreadsheet than write a computer program. Moreover, once written a program can often mask the mathematics that it is intended to represent, while on a spreadsheet the procedure is constantly exposed." (Steward 1994 p. 20)

Past research has noted that spreadsheets work particularly well in developing students' understanding of *algebra*. Generally speaking, creating and using spreadsheets "requires abstract reasoning by the learner" (Baker and Sugden 2007 p. 21) and *rule-making* (Vockell and van Deusen 1989) in order to perform operations on large ranges of cells. Keith Devlin provides an eloquent explanation as to why this process is a form of *algebra*:

> Doing algebra is a **way of thinking** … It is possible to do algebra without symbols, just as you can play an instrument without being able to read music… algebra involves thinking logically rather than numerically… in algebra, you introduce a term for an unknown number and reason logically to determine its value… With a spreadsheet, you don't need to do the arithmetic; the computer does it… What you, the person, have to do is create that spreadsheet in the first place… you need to think ***algebraically*** to set it up to do what you want. (Devlin 2011)

In using spreadsheets, students develop many of the key skills and understandings involved in algebra without getting bogged down in "moving around x's and y's": "Students look for patterns, construct algebraic expressions, generalize concepts, justify conjectures" (Friedlander 1998 p. 382). Since many students struggle to develop understanding through practicing symbolic manipulation, spreadsheets offer a much needed alternative approach to learning algebra (Sutherland 2007).

Along with algebra, spreadsheets are also well suited to teaching mathematical modeling. As Abramovich (2003) argues:

> The presence of technology in the teacher education classroom has great potential to enrich… modeling pedagogy by having teachers explore computer-enhanced models and formulate questions about those models … As far as a spreadsheet is concerned, its computational nature enables immediate feedback so that one can test emerging strategies and see results in ways that were never possible with more traditional, pencil and paper materials. (p. 2)

Due to the ability to repeatedly alter values and see the results almost instantly, spreadsheets encourage open-ended investigations and active learning (Beare, 1992). A great deal of research explains how spreadsheets can model problems in a wide range of areas: from finance (Benninga 1998, Sugden & Miller 2010) to chemistry (Billio, 1997) to astronomy (Whitmer, 1990).[3]

A final benefit worth discussing is the practical advantages spreadsheet skills have for students' future earnings. A recent study on the relation between technological skill and earning potential found:

> Eight in ten (78%) of middle-skill jobs demand facility with productivity software, and these digital jobs pay a premium over non-digital middle-skill roles. Additionally, productivity software is necessary for upward movement. Managerial roles across career areas, not only in offices but also in manufacturing and retail, rely heavily on word processing and spreadsheets (Burning Glass 2017, p. 9).

Further, Abramovich et al. (2010) note that spreadsheet capabilities may aid progress in more advanced STEM careers due to the high-level problem solving skills they develop.

3.1 Contrast with Alternatives

Granting that spreadsheets have these virtues, we should compare them with the technology most commonly used in the US high school classroom: the graphing calculator. As a result of its built-in functions, a spreadsheet can do virtually everything that can be done by a graphing calculator. In addition, it has key advantages, over the graphing calculator – *as it is traditionally used*.

- The table display on a computer screen makes it much more intuitive to use, since all stages of complex multipart calculations or models can be displayed at the same time.
- The ability to use a mouse and full keyboard is far less cumbersome than a calculator's keypad, given the additional keys available and the ability to use the mouse to move from one section of the table to another with one click, rather than scrolling across with the keypad.
- The ability to cross reference many different cells allows for much more powerful modeling.
- The ability to save worksheets, copy and paste, and import data from other sources, creates potential for much more in depth applications.
- The ability to import large data sets allows students to practice using the statistical methods taught in secondary education in a situation that calls for them, as opposed to the

[3] A comprehensive list of research on the possibilities for spreadsheet modeling is found in Abramovich (2003).

more contrived small data sets students are usually required to type into their calculators in order to practice finding single variable statistics such as measures of central tendency or measures of variability.

As Kissane (2007) notes, certain high-end calculators have a spreadsheet function built in that allows them to do most, if not all of these things. In so far as this is the case, the discussion can be seen as motivating the use of calculators' spreadsheet functionality in the classroom, rather than abandoning calculators entirely: currently, this feature is not part of typical calculator use in the high school classroom. In addition, using spreadsheets on calculators feels a little like reinventing the wheel: doing so in this medium is much less convenient than on a computer, given the smaller screen, less efficient input system, and lack of connectivity with other programs.

Despite this, spreadsheet software remains little used in mathematics education, while graphing calculators are prevalent. As Baker and Sugden (2007) note, it is plausible that the primary reason for this is logistical rather than educational. Most administrators have greater familiarity with calculators, and they are already easily accessible within most schools.

Calculators may be perceived as a cheaper option, though it should be noted that there now exist netbooks with spreadsheet capabilities of comparable price to high end graphing calculators. Though, admittedly, these calculators may easily be handed out and collected each lesson, as opposed to a more costly one-laptop per student policy. However, if, as we have argued, spreadsheets are the better technology for mathematics education, there is strong motivation to overcome these administrative hurdles.

3.2 Summary

This research presents compelling theoretical grounds to think that spreadsheets can be a valuable tool in mathematics education. In the following section, we will demonstrate how this can be manifested in the classroom through a lesson-series on retirement savings.

4. Modeling Retirement Savings

The example we will be working with is the task of modeling retirement savings in spreadsheets. This activity is designed for a high school algebra class and will take place over a series of 3-5 lessons. Students should have prior knowledge of exponents and be comfortable calculating present and future value using the compound interest formula. These lessons may be used to introduce students to geometric series, or to deepen their understanding of them if they have prior knowledge in this area.

These activities build on prior work by Sugden and Miller (2010), who provide a detailed analysis for performing financial calculations in Excel, including for annuities, similar to the retirement savings scenarios we are concerned with modeling. Their paper constitutes a comprehensive guide to performing virtually any compound interest problem one is likely to face in Excel, and is a valuable reference for anyone using the present lesson series.

We focus more narrowly on the specific problem of calculating future and present value for retirement savings. We also focus on what Sugden and Miller call the "linear recurrence method", in which all indices and terms in the series are input in the spreadsheet. While we agree with the authors that learning to solve the same problem using multiple methods is essential for developing mathematical understanding, and would be a worthy supplement to the activities we describe, we believe that the linear recurrence method draws upon the unique benefits of learning mathematics in Excel, and so is the method in need of extended discussion.

Our goal is to look in depth at the pedagogical issues surrounding this topic and make explicit how working with Excel in this context creates opportunities for building conceptual understanding in line with best practices in mathematics education. In part, we are concerned with outlining a case that will be compelling to mathematics teachers and mathematics education decision-makers to include spreadsheets in their curriculum.

The application of geometric series to retirement savings has a number of virtues. First, it is an *authentic* application of mathematics: calculating how much to save for retirement is a genuine problem most people will face, and the process employed in the lesson series matches the process that must be used in the real world. It does not need to be idealized or simplified. Second, this task incorporates a number of topics central to the study of algebra and allows students to explore the connections between them. Third, these topics are particularly well suited to be addressed using spreadsheet software: it is more effective than other mathematical tools in this context.

One might be concerned that retirement is not a topic high school students will find particularly relevant, given how far in their future it lies. We believe, though, that its richness and authenticity as a problem to model outweigh this concern. This is an area in which students can fruitfully incorporate their background knowledge about the financial world and relate it to personal experiences among their relatives, which allows for more effective learning.

In addition, it should be noted that much of what is discussed here can be applied to other financial problems that students may be facing imminently, such as repaying student loans, or an auto-loan. As part of the learning process, these topics can also be addressed – for example, as homework assignments. Modeling retirement, though, requires the most sophisticated modeling procedure and so serves as the natural end goal for a project on payment series.[4]

The starting point for this activity is to outline some background information on the nature of retirement savings. Most people want to retire upon reaching a certain age – stereotypically, the age of retirement is 65, though in reality this may vary significantly from person to person. At that point, they will need to sustain themselves without working. With no new source of income, they must have saved enough money by the time they retire to cover their expenses through the remainder of their life.

With this in mind, the project can be understood by working backwards. The central question is: How much should one save to meet one's retirement goals? Background information and various concepts must be specified and understood in order to answer this question:

- A person makes annual investments for retirement over the course of their career.
- They expect to retire at a certain age and live for a certain number of years after retirement.
- When they retire, their goal is to have a certain percentage of their salary at the point of retirement as their retirement income.
- The investments earn interest.
- The amount invested will grow each year in proportion to the increase in salary over the course of their career.
- Upon retirement, the amount saved will continue to earn interest, but typically at a lower rate than before, as it will be transferred to lower risk investments.
- There is inflation over the course of the investment.
- The person's retirement income should grow over the course of their career in line with inflation.

These issues should be discovered or drawn out through discussion with students about saving for retirement, as they flow naturally from reflection on the needs of someone planning for retirement. This is a consequence of the task being an *authentic* application. Thinking critically about what the scenario entails will enable students to come up with the parameters for the model.

A skillful teacher will structure the discussion so students make these discoveries, drawing on their background knowledge of financial life; however, in some situations, the teacher may have to specify certain features of the scenario themselves or draw attention to financial points the students are not aware of. Research shows that learning through discovery,

[4] See [Author 2] for details on a broader sequence of activities involving payment series.

even when teacher assisted, is more effective than learning through a traditional lecture format, which is why the authenticity of the application is crucial (Brown, Roediger & McDaniel, 2014).

Pulling from best practices in project-based learning, teachers can treat addressing the problem of saving for retirement as a project that motivates the trajectory of the lessons (Thomas, 2000). The discussion of issues involved in saving should take place at the start of the lesson-series, and students should keep the central question in mind as the goal they are building towards. With some classes, it may be appropriate to *scaffold* the problem by introducing a few assumptions at a time. For example, the teacher could hold off discussing the concept of inflation until later in the course.

While the question of how much one should save for retirement can be addressed through more traditional methods, modeling it within a spreadsheet is an attractive option. The spreadsheet free method requires using an ungainly and impractical formula for future value: $FV = \frac{c}{r-g}[1 - \frac{(1+g)^n}{(1+r)^n}](\frac{1+r}{1+i})^n$.[5] Avoiding this formula can serve as a motivator for students to employ spreadsheets. After being given both methods, teachers report that students prefer to work with spreadsheets.[6] It also provides students with a sense of achievement to create a spreadsheet that does the same work as such an intimidating looking formula.

When beginning this project, the first point for students to learn mathematically is that the future value of a series of investments of equal value made at regular intervals forms a geometric series.[7] In preparation for spreadsheet work, students must work with geometric series in a 'table' format. Instead of listing the terms horizontally, as is often done in secondary textbooks, they should be displayed in a table, with the indices in one column and the values in the next. This is the same format used in a spreadsheet model, as illustrated in *Figure 1*:

[5] See [Author 2]. Formula key: FV=Future Value; c=first payment; g=growth rate; r=discount rate; n=length of investment.

[6] See section 5 of this paper and [Author 1].

[7] For example, if you invest $1000 per year for ten years with an 8% interest rate, your first investment will earn interest for 10 years, your second investment will earn interest for 9 years, while your final investment will earn interest for only one year. Therefore, your series of investments (counting from last to first) will have future values: 1000·1.08, 1000·1.082, 1000·1.083,...1000·1.0810. This is a geometric sequence, with first term 1000·1.08, common ratio, 1.08, and 10 terms. The total value of your investment is the geometric *series* obtained by summing this sequence. See [Author 2].

	A	B	C	D	E
1	**Index**	**Recursive**		**Formula**	
2	1	6		6	
3	2	18	=B2*3	18	=6^(A3-1)
4	3	54	=B3*3	54	=6^(A4-1)
5	4	162	=B4*3	162	=6^(A5-1)
6	5	486	=B5*3	486	=6^(A6-1)
7	6	1458	...	1458	...
8	7	4374		4374	
9	8	13122		13122	
10	9	39366		39366	
11	10	118098		118098	
12	11	354294		354294	
13	12	1062882		1062882	
14	13	3188646		3188646	
15	14	9565938		9565938	
16	15	28697814		28697814	

Figure 1

The image is taken from a spreadsheet modeling a geometric sequence with first term 6, common ratio 3, and 15 terms. This presentation fosters mathematical reasoning and is suitable for introducing students to geometric series for the first time or deepening the understanding of students who are already familiar with them. The table format provides a new perspective on sequences by emphasizing *indices*: rather than referring to 'the nth term' in a sequence, and using the subscript notation $a_1, a_2 \dots a_n$ that many students find confusing, students can think in terms of 'the value in row n', which is intuitively displayed in the data.

This format gives a tangible representation of the recursive and explicit formulas for the sequence, when used in a spreadsheet. Since the recursive formula refers to the previous term in the sequence, the Excel function must reference the cell *above*. On the other hand, the explicit function only uses the index, so it must reference the cell to the *left*. Examples of such formulas are displayed in *Figure 1*.

This presentation of the explicit formula also demonstrates the connection between a sequence and a function. A sequence is equivalent to a function from the natural numbers to the real numbers.[8] The index column specifies the domain of such a function, while the values displayed in the formula column specifies the range. In addition, the function entered in the formula column specifies the *rule* that takes you from the domain to the range.

[8] See, e.g., Rudin (2006).

This approach to functions exhibits Devlin's conception of algebra, mentioned above. When thinking about the relationship between domain and range, it is clear there is a connection between two families of numbers – visually represented in separate columns. In addition, the function used to connect the two is demonstrated in a tangible way by referencing the relevant column. When using the fill feature, it is made clear how the same rule is applied to all the entries in the domain in turn. Together, this provides students with a new way of thinking in general terms about how numbers are related. For those who struggle with the traditional 'symbol manipulation' approach, this may better suit their learning style. For kinesthetic learners, it's also possible that the act of using the fill function and physically dragging to apply the rule to those digital cells and watching the values appear in real time could help solidify this connection.

A final advantage, from a practical perspective, is that students can also work with much larger series than is otherwise feasible: a 100-term series can be created in seconds. Provided students are competent with arithmetic and do not need additional practice, this allows them to skip the tedious task of creating each term of the sequence and get to the mathematically interesting part of looking at entire series and analyzing them. The immediacy and ease of this better communicates the non-terminating nature of a sequence and encourages exploration of multiple orders of magnitude by removing time-related and computational barriers.

For all these reasons, modeling geometric sequences with spreadsheets encourages mathematical reasoning and sense making with regard to concepts in algebra, which we established above is required for effective use of technology.

With the mathematical fundamentals in place, students can move on to modeling retirement savings. For a simple payment series, like the one discussed above, the process is exactly the same as with modeling any other retirement series. For a more realistic model, though, additional parameters come into consideration. The value of the annual payment grows over time, which must be factored in, along with interest earned. In addition, there is a target sum – the value that must be received to meet the retirement goal. *Figure 2* displays how all this information can be calculated and presented in a spreadsheet:

	A	B	C	D	E	F	G
1	**Assumptions**		**Payment no.**	**Growth Factor**	**Payment**	**FV Factor**	**FV**
2	**First Payment**	$ 3,500.00	1	1.00	$ 3,500.00	3.95	$ 13,830.21
3	**Growth rate**	5%	2	1.05	$ 3,675.00	3.68	$ 13,508.58
4	**Interest rate**	8%	3	1.10	$ 3,858.75	3.42	$ 13,194.43
5	**Term**	20	4	1.16	$ 4,051.69	3.18	$ 12,887.58
6			5	1.22	$ 4,254.27	2.96	$ 12,587.87
7			6	1.28	$ 4,466.99	2.75	$ 12,295.13
8	**Results**		7	1.34	$ 4,690.33	2.56	$ 12,009.19
9	FV (Direct Calculation)	**$ 223,237.48**	8	1.41	$ 4,924.85	2.38	$ 11,729.91
10			9	1.48	$ 5,171.09	2.22	$ 11,457.12
11			10	1.55	$ 5,429.65	2.06	$ 11,190.68
12			11	1.63	$ 5,701.13	1.92	$ 10,930.43
13			12	1.71	$ 5,986.19	1.78	$ 10,676.23
14			13	1.80	$ 6,285.50	1.66	$ 10,427.95
15			14	1.89	$ 6,599.77	1.54	$ 10,185.44
16			15	1.98	$ 6,929.76	1.44	$ 9,948.57
17			16	2.08	$ 7,276.25	1.34	$ 9,717.21
18			17	2.18	$ 7,640.06	1.24	$ 9,491.22
19			18	2.29	$ 8,022.06	1.16	$ 9,270.50
20			19	2.41	$ 8,423.17	1.08	$ 9,054.90
21			20	2.53	$ 8,844.33	1.00	$ 8,844.33
22			**Total**				**$ 223,237.48**

Figure 2

This model introduces some significant new features in addition to those discussed in modeling geometric series in general. First, the function calculating future value is *broken down* into stages. This allows students to think about how a complex function can be decomposed into simpler sub-functions. They can also analyze the effects of the different components. For example, 'FV factor' demonstrates the effect of interest – showing how much a dollar at the time of investment is worth at the term of the payment series. This promotes understanding of exponential growth through an authentic application.[9]

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[9] For a series of investments, with first payment *c*, growth rate *g*, interest rate *r*, and term *n*, the future value for investment number *t* is given by the formula: FV=c(1+g)t-1(1+r)n-t. To analyze the components:

1) (1+g)t-1 is called the growth factor, for every $1 you invest with the first payment, in year t you invest $(1+g)t
2) The value of payment *t* (at the time of the payment) is first payment • growth factor.
3) (1+r)n-t is call the FV factor, it tells you how much a dollar at the time of payment is worth at the term of the investment.

FV= c • (1+g)t-1 •(1+r)n-t

FV=First Payment•Growth Factor•FV Factor

See [Author 2].

Giving students an intuitive, visual method for breaking down a complex function into its simpler constituents is an invaluable skill, especially when it comes to mathematical perseverance (NCTM 2014). An essential component of perseverance is the ability, when presented with something complicated and unfamiliar, to break it down into smaller parts that one has experience dealing with, and work with them in isolation before looking at how they fit together.

In addition, this process allows students (or teachers) to efficiently troubleshoot by quickly identifying the column in which there is an error, rather than having to work through the whole function operation by operation. This is in contrast to the often painstaking process of working out what went wrong when a complex input into a calculator yielded an incorrect result. Again, this allows the focus to remain on mathematical reasoning, rather than getting held up correcting a trivial typing mistake.

Another feature of note is the use of absolute and relative reference. When a cell is referred to using absolute reference, the cell referenced does not change as the formula is filled into new cells, the way it does with relative reference.[10] Absolute reference is indicated by dollar sign: for example, if the input in cell B1 is '=A1+1', this makes absolute reference to the cell A1. This means if one fills down or across, the input in the filled cells will be the same (in B2, for example, the input will still be '=$A1$+1'). However, if the input in B1 is '=A1+1', the reference to A1 is *relative*, so the cell reference changes when one fills down or across (in B2, for example, the input will be '=A2+1'). Absolute reference is needed for the 'assumptions' in *figure 2*, as they don't change from payment to payment.

This illustrates the distinction between a constant and variable. In traditional algebra, it is very easy to confuse a variable with a particular kind of *symbol*. Students often think a variable is simply a 'letter in an equation', but this is also true of constants and unknowns, which can lead to students conflating these mathematically distinct categories. In contrasting absolute and relative reference, it is clear this is not the case. When students experiment with the fill feature with either absolute or relative reference, they get a tangible experience of how the reference *functions* – whether it varies across a domain or remains fixed.[11] This provides another instance in which spreadsheets are well suited to help build conceptual understanding, rather than mere procedural knowledge.

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[10] See Abraham et al (2008).

[11] See Lagrange & Erdogan (2009) p.73: "absolute references correspond to the mathematical idea of parameter, whereas relative references correspond to variables and thus using both kinds of references is linked to the idea of literals having different algebraic status."

It should be noted that when using spreadsheets optimally, the constants and variables should be *named* (Baker & Baker 2010). This would remove the need to use the absolute reference symbols in the formulas discussed above – for example, the entry in D2 of figure 2, would be something like "=growth*number". Though the topic of naming is an essential part of spreadsheet use, it need not be introduced within this lesson series if students are not yet familiar with it. The reason for this is two-fold: first, as discussed, learning to use absolute reference develops valuable mathematical understanding surrounding variables and constants; second, learning how to create and manage names in Excel is the kind of platform specific procedural learning that may create an impediment to students doing mathematics. Of course, naming enhances the ability to do mathematics in Excel in the long run and students should master it at some point in their education; we believe it should be down to the discretion of the educator precisely when to introduce students to this topic.

Once students have created this model, they are able to interact with it to see how changing parameters changes the output. Spreadsheets are ideally suited for this kind of exercise, as work can be saved and copied and any cell can be changed with a mouse click. As a possible activity here, the initial set up should be arranged so that the person is not on track to meet their retirement goals. Students can alter the variables in the model to see what options are available. Because the application is authentic, it allows for critical analysis. The students need to think carefully about what changes to the model imply about the real world scenario, and they must offer a solution which is realistically achievable. For example, they could make the numbers work by increasing the interest earned on savings or increasing the person's salary; however, these are not changes that are usually within a person's power to implement. On the other hand, increasing the rate of savings or the age of retirement are measures a person could realistically take.

In summary, there is good theoretical reason to think that use of spreadsheets in this project deepens students' understanding of algebra, as well providing an effective model of a real world scenario. Thus, it is line with best practices in mathematics education for effective use of technology.

5. Qualitative Research

In order to test our ideas, we have run a series of workshops instructing mathematics teachers on how to use this activity with their students. All participants, despite some having little prior familiarity with spreadsheets, were able to model growing cash flows by the end of the workshop. This confirmed our belief that spreadsheets provide an intuitive vehicle for practicing mathematical reasoning.

In addition, we solicited feedback from participants to gauge the effectiveness of the activity. This feedback informed us that teachers' views on the subject were broadly in agreement with our own. Over 90% of participants agreed with the statements: "I plan to continue working on an idea/strategy from this course," and "I will share what I learned in this course with another teacher or colleague."

Below are some specific comments we received that show how participants' experiences were in line with our aims for the lesson series:

- "By using Excel to show how arithmetic and geometric sequences and series are calculated, it is a powerful tool to quickly show my students how time and rate affect investment totals."
- "I gained a lot of knowledge about Excel and how to better break down my formulas and present Excel work to students as they're learning."
- "It was interesting to apply Excel worksheets to analyze more complex and real world mathematical topics and make them more accessible to all students."
- "Using the Excel spread sheet to do the heavy-duty calculations can make learning real world math problems a bit easier."
- "The use of spreadsheets to demonstrate what happens over time rather than using formulas provides a different lens through which to view math."
- "I will share with my colleagues the fact that we don't always need to focus our teaching on formulas. Much of the most important connections and understandings in math are not revealed simply by plugging into a formula."
- "I will share this not only with my fellow math colleagues but also anyone interested in the time value of money. It's a great tool to show the power of the exponential function in regards to growth."

In the future, we hope to expand these workshops and collect data that allows us to make a *quantitative assessment* of their effectiveness.

Conclusion and Recommendations

We have demonstrated the benefits of and reasons for incorporating the use of spreadsheets in mathematics education. In addition, we have provided an example of how to do so in accordance with best practices in mathematics education. Qualitative research on our model case confirms our analysis.

Despite this, we noted above that the use of spreadsheets in US schools is far less frequent than the use of graphing calculators. With the emergence of low price 'netbooks' and other low-cost laptops, this is not due to any insurmountable material barrier. A potential obstacle here is the issue of testing. Most state tests, as well as the SATs and other standardized assessments, require the use of a graphing calculator and forbid the use of a computer or any technology equipped with CAS (a computer algebra system). Students must learn to use the technology that will be made available to them in these important benchmark exams that can be very important and influential in determining their educational and professional future. This paper provides motivation for pushing to change the regulations for mathematics testing. If these assessments are to test the mathematical skills that matter, they should test how well students can work with spreadsheets. Doubtless, the inertia against such dramatic change is strong, so the project will take time and effort. In the meantime, there is good reason for mathematics education to employ spreadsheets *in addition* to graphing calculators.

Two key future tasks arise: first, creating a comprehensive set of mathematical spreadsheet activities to be incorporated into a mathematics curriculum; second, researching student learning when being taught mathematics through spreadsheets. The aim of this paper has been to make the case that this further research is a worthwhile endeavor.

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