FiCycle Working
Paper Series

FiCycle Standards for Finance and Mathematics

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Our overarching goal at FiCycle is to promote high-quality finance and mathematics education. In deciding how to do this we are guided by five key principles:

1. Students learn best when they develop conceptual understanding.
2. A conceptual understanding of personal finance requires connecting learning to the underlying mathematics.
3. Student learning is enhanced when they find the material interesting and relevant to their lives.
4. Many students who are disengaged with pure mathematics find applications to finance relevant and interesting.
5. Student and teacher time is a scarce and valuable resource.

In this paper, we elaborate on these principals and show how they lead us to the FiCycle standards for a course in the mathematics of personal finance. We discuss the financial and mathematical theory behind these standards, before laying them out in full at the end of the document.

Our Principles

We believe it is essential for students to receive a high-quality education in mathematics and a high-quality education in personal finance. Further, we believe that offering students a rigorous mathematics course which uses financial topics for all of its applications advances both of these goals. In this document, we present the theory and research supporting these beliefs and outline a set of standards for a course in mathematics and finance in line consistent with the theory and research. The Financial Life Cycle Mathematics course has been designed to meet these standards, but we also believe a whole range of high school mathematics courses could be created that do so in a variety of fruitful ways.

Our approach rests on the following key principles:

P1. Students learn best when they develop conceptual understanding.
P2. A conceptual understanding of personal finance requires connecting learning to the underlying mathematics.
P3. Student learning is enhanced when they find the material interesting and relevant to their lives.
P4. Many students who are disengaged with pure mathematics find applications to finance relevant and interesting.
P5. Student and teacher time is a scarce and valuable resource.

Principles 1 and 2 explain why we believe that teaching finance within a mathematics course improves the quality of finance education. Principles 3 and 4 explain why we believe basing a mathematics course around applications to finance improves the quality of mathematics education. Principle 5 is, in effect, a cautionary note that guides our thinking: it says that introducing a new course into the curriculum can come with a huge opportunity cost, since students will have to eliminate one course from their schedule to make room for another. We believe that replacing a traditional final year mathematics course with a mathematical finance course is uniquely well positioned to justify this cost, as students will retain the mathematics training while gaining additional benefits through the financial applications.

In what follows, we outline the justification for these principles.

Understanding Personal Finance

A range of research shows that young people across the US are in a precarious financial situation and lack essential financial knowledge. Further, inadequate financial knowledge is closely connected to undesirable financial outcomes, such as burdensome debt and insufficient savings.

Financial education has been proposed as a solution to this pressing problem. Though such education programs have been shown, on the whole, to improve financial knowledge, their effectiveness varies

massively. It is important, therefore, to better understand the best practices that lead to effective education programs. We discuss this further in our academic literature review.[1]

A key finding in education research is that effective learning requires conceptual understanding. Being taught a series of procedures for completing a list of rote tasks, without addressing the connections between them, leads students to learn little and struggle to remember what they do learn. Students learn and retain knowledge much better when they grasp the fundamental concepts connecting the different content areas. Since one key goal of financial education is for students to retain what they learn and apply it to important financial decisions in adulthood, it is particularly important that financial education be designed to promote knowledge retention.

To achieve this goal, a high quality education in finance should ensure that students develop a conceptual understanding of the subject matter. In line with academic financial theory, we believe that conceptual understanding of personal finance requires a grasp of the financial life cycle. Specifically it requires:

- Understanding how and why to transfer consumption over time.
- Understanding how and why to manage risk.

In order to achieve this, an excellent financial education must include these four essential understandings:

E1. The fundamental measure of financial wellbeing is wealth, and financial statements can be used to measure and manage wealth over the financial life cycle.
E2. Consumption is transferred forward and backward over time using payment series with compound interest. Understanding payment series requires applying the mathematical concepts of exponents and geometric series.
E3. Risk can be measured using probability and expected utility. This provides the means to evaluate risk management tools such as insurance.
E4. Investments, including stocks, can be modelled as probability distributions. These distributions can be used to calculate the risk of investments.

These essential understandings require grasping a range of mathematical concepts and relating them to the relevant financial principles. Therefore, best practices require integrating financial education with mathematics education.

[1] See FiCycle (2021) 'FiCycle Research Overview'.

Making Mathematics Relevant

Math achievement in the US is insufficient, especially at the high school level. US students consistently rank poorly compared to those in other developed countries. Further, research demonstrates that there is a strong link between mathematics achievements and economic outcomes. In particular, high levels of mathematics education are associated with high income levels.

A significant problem in education is student disengagement. In math class in particular, it's often reported that students are uninterested in the materials and don't view it as related to 'real life'. It has been shown, however, that learning is much more effective when students are highly engaged. One way to improve engagement is through attention to the material being taught. Studies show improved results when students find the material relevant and interesting.

Mathematics leaves much room for improvement in this regard. Many so-called real world applications of math are contrived word problems, where supposed application is irrelevant to the solution of the problem. What's needed are authentic applications, where the mathematical processes can actually be used to guide decision-making.

We believe that applications to finance are well-suited for this purpose. High school level mathematics facilitates understanding and addressing many financial problems that are relevant to students making for an authentic and interesting application. Therefore, integrating financial applications into a mathematics course will improve mathematics education by increasing engagement.

Opportunity Cost

Recall our fifth principle: "Student and teacher time is a scarce and valuable resource." There are only so many hours in the school year, both for students to learn and for teachers to teach; all of these hours are already in use. Therefore, adding something to the curriculum requires removing something else. To justify such an addition, it must be shown that the benefits of adding the new course outweigh the costs of removing the old one.

This is a prominent problem for alternative approaches to financial education that recommend implementing a standalone course in personal finance. To do this, some course in an unrelated subject matter must be eliminated – from the arts, sciences, or humanities. For example, a new financial education initiative in North Carolina required cutting the high school American history coursework in

half in order to accommodate the personal finance requirement. Arguing that the benefits here clearly outweigh the costs is a tough case to make.

We also question the efficacy of the "no cost option" of making finance education an extra-curricular course. Though this appears to offer the benefits of financial education without any cost, since nothing has to be cut, the appearance is deceptive. First, students participating in one extra-curricular activity likely reduces their participation in another, and there are all many benefits to students participating in sports or creative arts, among other things. Second, an extra-curricular course is likely to be shorter and less rigorous than one taught within regular school hours. Research shows that such less rigorous courses are less effective.

Our approach, on the other hand, does not require eliminating a course, but instead changing the focus of one high school mathematics course such as a traditional Algebra 2 course. In this case, the opportunity cost arises from the change in focus of the mathematics education and the likelihood that the financial mathematics course covers fewer mathematical topics since time must also be set aside to discuss the financial concepts and applications.

We do not believe these costs are too severe, based on the previous discussion of best practices in mathematics education. Beyond mastering arithmetic, the significance of mathematics education lies primarily in developing mathematical *reasoning*, rather than getting students acquainted with as many mathematical concepts as possible: quality rather than quantity is our goal. Not, as current practice is often described, a curriculum that is "a mile wide and an inch deep." If, as we argue, spending time exploring mathematical applications to finance increases students' engagement and fosters a deeper understanding of mathematical concepts, the tradeoff is justified.

Unpacking the Principles

These ideas lead us to our standards for a course in finance and mathematics. As we discussed in section 2, we believe than an excellent personal financial education should provide understanding of the financial and mathematical concepts necessary to navigate the financial life cycle.

For a person to achieve good financial outcomes, they must have the financial means to meet their goals across their whole lifetime. One's financial needs and abilities vary over the course of one's life cycle, and one must plan for this when making financial decisions; this is what we mean by 'navigating the financial life cycle'. To do this requires:

Understanding how and why to transfer consumption over time.

- One's income varies across one's lifetime, as do one's necessary expenses, but they do not always line up, so one must use financial products to make sure one has the necessary resources available at different points in time to meet these needs.

Understanding how and why to manage risk.

- One's future, financially speaking, is uncertain, and some possible outcomes would be incredibly unpleasant if sufficient financial resources were unavailable. Financial products can be used to ensure that the necessary resources will be available across a wide range of outcomes.

As we discussed, fully grasping these ideas requires four essential understandings.

E1: The fundamental measure of financial wellbeing is wealth, and achieving such wellbeing requires measuring and managing wealth over the financial life cycle.

- In order to think about navigating the financial life cycle, we need to know how to adequately measure a financial situation. This requires thinking in terms of wealth, rather than cash balance. Students must understand that their wealth is the means by which they can meet their needs across the life cycle.

E2: Consumption is transferred forward and backward over time using payment series with compound interest. Understanding payment series requires applying the mathematical concepts of exponents and geometric series.

- All borrowing and investing involves compound interest, which makes understanding how to transfer consumption inseparable from a mastery of the mathematics of exponents and functions. Further, in most significant real life instances of transferring consumption, payments will be made at regular intervals, rather than in lump sum. That means students must move beyond the simple calculations of compound interest that are typically taught to working with interest within geometric series.

E3: Risk can be measured using probability and expected utility. This provides the means to evaluate risk management tools such as insurance.

- The fundamental concept when making any kind of decision under uncertainty is expected utility, and this includes financial decisions. To understand this, students must master the mathematics of probability and expected value. Combining the mathematical tools with a critical

analysis of uncertain financial situations allows them to properly understand the importance of insurance.

E4: Investments, including stocks, can be modelled as probability distributions. These distributions can be used to calculate the risk of investments.

- Building significant wealth in the long term generally requires investing in the stock market, so it is essential for students to understand how to approach this in an informed manner.
- Mathematical analysis shows that for diversified stock market portfolios, over a sufficient period of time, the expected return is likely to exceed the incremental risk. Conversely, short term undiversified investments have more variable outcomes.

Based on this elaboration of the key essential understandings, we have created a list of standards for a course in personal finance and mathematics. They are broken down in into finance and mathematics standards and grouped according to essential understanding.

The FiCycle Standards

<table>
<tr><td colspan="2">E1: The fundamental measure of financial wellbeing is wealth, and achieving such wellbeing requires measuring and managing wealth over the financial life cycle.</td></tr>
<tr><td>Finance Standards</td><td>Mathematics Standards</td></tr>
<tr><td>F1.1 Students know that wealth is defined as ability for consumption, and understand its significance over and above cash balance.</td><td>M1.1 Students are able to manipulate and substitute linear equations, and reason with systems of equations, including key financial applications:

a. The net worth equation: $NW = A - L$ (net worth = assets - liabilities.
b. The net income equation: $NI = I - E$ (net income = gross income – expenses)</td></tr>
<tr><td>F1.2 Students understand the four key concepts for measuring wealth: income, expense, asset and liability.

a. Students are able to track and calculate these values using financial statements: income statements, balance sheets, and budgeting tables.</td><td>M1.2 Students understand the relationship between an equation and a function taking elements of the equation as inputs. They apply this when relating financial equations to financial statements. (For example, they create a function that gives net worth as an output using the asset and liabilities entries on a balance sheet as inputs.)</td></tr>
<tr><td rowspan="2">F1.3 Students understand the key factors that influence wealth:

a. The connections between income level, career and education.
b. The difference between incurring an expense and purchasing an asset.
c. The importance of maintaining a cash balance to preserve liquidity for emergencies.
d. The relationship between taxes and income.</td><td>M1.3 Students are comfortable with units and percentages when dealing with currency and taxes, respectively.</td></tr>
<tr><td>M1.4 (*) Students understand piecewise functions and use them to model marginal tax rate, total tax, and effective tax rate.</td></tr>
<tr><td colspan="2">E2: Consumption is transferred forward and backward over time using payment series with compound interest. Understanding payment series requires applying the mathematical concepts of exponents and geometric series.</td></tr>
</table>

F2.1 Students understand the role of borrowing and investing in transferring consumption across the financial life cycle. a. If one has a surplus of income now, one can invest it in order to use it at future time when one's consumption needs are higher than one's income. b. If one's current expenses exceed one's current income, one can borrow money to meet them and repay the money with future surplus income.	M2.1 Students understand the rules of exponents, including negative exponents, and are comfortable manipulating them in algebraic expressions. Understand and employ the compound interest formula $FV = PV \bullet (1 + \frac{r}{n})^{n \cdot t}$ Understand and use the discounting formula $PV = FV \bullet (1 + \frac{r}{n})^{-n \cdot t}$ a. They understand the discounting formula's relationship to the compound interested formula.
F2.2 Students understand that the value of money changes over time due to interest: a dollar today is worth more than a dollar in the future. a. Interest is earned or paid as a percentage of the value being transferred. b. The connection between present value and future value is calculated using the compound interest equation.	M2.2 Students understand Euler's number, and use it in the continuous compounding formula $FV = PV \cdot e^{rt}$ a. (*) Students understand the definition of Euler's number as a limit and use this to derive the continuous compounding formula.
F2.3 Students know the different borrowing needs one may face and the financial instruments for meeting those needs. a. Buying a house builds wealth, through spending money on housing equity, an asset, rather than rent, an expense. b. A mortgage is a loan used for buying a house. c. Spending money on college gives one a qualification, degree, or skills that can lead to a future career with a higher income. d. Student loans are used to cover the costs of education.	M2.3 Students the importance of estimation and how to employ it effectively. Students apply this to using the rule of 72 to estimate the time it takes for an investment of debt to double in value. $t = \frac{72}{R \cdot 100}$ a. (*) Students understand the rules for logarithms, including natural logs, and use this to derive the rule of 72.

e. A credit card can be used to borrow small amounts of money instantly but comes with a high interest rate. f. Credit scores determine one's access to a variety of borrowing opportunities and are determined by past financial behavior.	
F2.4 Students know the different investment needs one may have and the financial instruments for meeting those needs. a. Savings accounts allow one to invest money with very low risk, high flexibility and low interest rates. b. Treasury Bonds are a tool for borrowing with low risk, low flexibility, and modest interest rates. c. Stocks are higher risk investments with higher returns on average. d. Taking out a mortgage to buy a home requires making a down payment. e. Upon retiring, one will no longer have income to meet one's living expenses, so one must invest for this while working. f. One may have to deal with unexpected expenses which one should save in preparation for.	M2.4 Students understand how to calculate and model with arithmetic and geometric series and apply them to payment series appropriately. a. Series of simple interest payments can be modeled with an arithmetic series. b. Series of compound interest payments can be modeled with a geometric series. c. Series can be created using recursive and explicit formulas for sequences. d. When modeling realistic financial instruments and cash flows, one must add additional variables for growth rate and inflation.
F2.5 Students know that investments and repayments typically occur through a series of payments over an extended period of time and understand how this affects financial decisions. They can understand situations involving the following features: a. Annuities and amortization b. Growing Payment series c. Inflation	M2.5 Students understand how to breakdown complex formulas into simpler constituents. They can apply this to complex payment series formulas.
E3: Risk can be measured using probability, expected value, and expected utility. This provides the means to evaluate risk management tools such as insurance.	

F3.1 Students understand the different kinds of financial risk one faces over the financial life cycle: a. Healthcare costs b. Vehicle damage c. Property damage/theft d. Device damage/malfunction e. Dependent impoverishment due to death of care giver	M3.1 Students understand the fundamental features of probability and use this to measure financial risk. a. $P(Sample\ Space) = 1$ b. $P(E) = \frac{\#(Outcomes\ in\ which\ E\ occurs)}{\#(Outcomes\ in\ the\ experiment)}$ c. If *a* and *b* are independent events, then $P(a\&b) = P(a) \cdot P(b)$
F3.2 Students understand how insurance mitigates risk and how the nature of the risk affects the need to purchase insurance. a. With insurance, you pay someone else to take on risk for you. b. Diversified vs correlated risk c. Catastrophic risk, and ability to cover with savings.	M3.2 Students understand the concepts of expected value and expected utility and apply them to financial decisions. Expected Value Formula: $EV = \sum_{i=1}^{n} P(o_i) \bullet V(o_i)$ Expected Utility Formula: $EU = \sum_{i=1}^{n} P(o_i) \bullet U(o_i)$
F3.3 Students know about the different kinds of insurance, their distinguishing features, and the vocabulary that surround them. a. Premium, deducible, copay, limit b. Health insurance c. Car insurance d. Home insurance e. Device insurance f. Life insurance	M3.3 Students understand binomial distributions and can calculate an outcome's probability in binomial experiments. They are able to use this to model situations with diversified risk. a. Binomial Theorem: $P\ (k\ sucesses) = (n\ k\ )p^k(1\ -\ p)^{n-k}$ b. Binomial experiments can be represented by binomial trees
E4. Investments, including stocks, can be modelled as probability distributions. These distributions can be used to calculate the risk of investments.	

F4.1 Students understand the nature of investments in equities and the specific concepts of shares, dividends and returns.	M4.1 Students can model stocks as random walks and understand how this is a form of binomial distribution.
F4.2 Students understand the Efficient Market Hypothesis, and the implications it has for investment decisions.	M4.2 Students understand the concepts of mean and standard deviation and can relate this to financial risk.
F4.3 Students understand the relationship between risk and return: that investors are generally risk adverse, and so higher risk investments tend to come with a higher expected return.	M4.3 Students understand the key features of the normal distribution, and the situations in which the binomial distribution approximates the normal distribution.
F4.4 Students relate their knowledge of stocks and risk to their understanding of the financial life cycle in order to evaluate when investing in stocks is and is not appropriate.	M4.4 Students can mathematically model investments with different portfolio sizes over different lengths of time and observe general patterns between these factors and financial risk.