Combining Mathematics and Finance Education: Research Overview

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We believe that taking a course combining high school algebra with personal finance will improve the quality of students' mathematics education and their financial education. We believe that mathematical knowledge and financial knowledge are mutually reinforcing. Our goal is to spread the implementation of such courses throughout the US to increase mathematical understanding and financial well-being.

In this paper, we present the story so far, including our own research investigating the effectiveness of a math and finance course. Additionally, we outline the future work needed to best make the case for such a course and understand how to implement it most effectively.

Problem Statement

A wealth of research tells us that there is a pressing need to improve levels of both mathematical and financial knowledge in the US. 40% of Americans don't have $400 to cover emergency expenses (Economic Well-being report, Federal Reserve Board 2018). The median retirement savings for Americans age 55-64 was $107,000, roughly one-quarter of what experts recommend (Government Account Office 2017). In addition, fewer than 50% of U.S. high school graduates in 2016 were considered ready for college-level mathematics work, as measured by ACT mathematics scores (ACT 2016).

In both cases, education programs have been proposed as part of the solution. A range of financial education courses have been instated to try and improve financial knowledge and as a result improve financial outcomes. In addition, efforts by influential education organizations, such as the National Council of Teachers of Mathematics (NCTM), have attempted to reform standards and practices for mathematics education in order to improve results. We believe that these two projects work best if they are combined.

Our Principles

We believe it is essential for students to receive a high-quality education in mathematics and a high-quality education in personal finance. Further, we believe that offering students a rigorous mathematics course which uses financial topics for all of its applications advances both of these goals. In this document, we present the theory and research supporting these beliefs and outline a set of standards for a course in mathematics and finance consistent with the theory and research.

Our approach rests on the following key principles:

P1. Students learn best when they develop conceptual understanding.
P2. A conceptual understanding of personal finance requires connecting learning to the underlying mathematics.
P3. Student learning is enhanced when they find the material interesting and relevant to their lives.
P4. Many students who are disengaged with pure mathematics find applications to finance relevant and interesting.
P5. Student and teacher time is a scarce and valuable resource.

Principles 1 and 2 explain why we believe that teaching finance within a mathematics course improves the quality of finance education. Principles 3 and 4 explain why we believe basing a mathematics course around applications to finance improves the quality of mathematics education. Principle 5 is, in effect, a cautionary note that guides our thinking: it says that introducing a new course into the curriculum can come with a huge opportunity cost, since students will have to eliminate one course from their schedule to make room for another. We believe that replacing a traditional final year mathematics course with a mathematical finance course is uniquely well positioned to justify this cost, as students will retain the mathematics training while gaining additional benefits through the financial applications.

In what follows, we outline the justification for these principles.

Understanding Personal Finance

A range of research shows that young people across the US are in a precarious financial situation and lack essential financial knowledge. Further, inadequate financial knowledge is closely connected to undesirable financial outcomes, such as burdensome debt and insufficient savings.

Financial education has been proposed as a solution to this pressing problem. Though such education programs have been shown, on the whole, to improve financial knowledge, their effectiveness varies greatly (Kaiser & Menkoff 2017, 2018). It is important, therefore, to better understand the best practices that lead to effective education programs. We discuss this further in our paper "Best Practices in Financial Education: Incorporating Mathematics".[1]

A key finding in education research is that effective learning requires conceptual understanding. Being taught a series of procedures for completing a list of rote tasks, without addressing the connections between them, leads students to learn little and struggle to remember what they do learn. Students learn and retain knowledge much better when they grasp the fundamental concepts connecting the different content areas (Brown, Roediger, & McDaniel 2014). Since one key goal of financial education is for students to retain what they learn and apply it to important financial decisions in adulthood, it is particularly important that financial education be designed to promote knowledge retention.

To achieve this goal, a high-quality education in finance should ensure that students develop a conceptual understanding of the subject matter. In line with academic financial theory, we believe that conceptual understanding of personal finance requires a grasp of the financial life cycle (Ando & Modigliani 1963). Specifically:

- Understanding how and why to transfer consumption over time.
- Understanding how and why to manage risk.

In order to achieve this, an excellent financial education must include these four essential understandings:

E1. The fundamental measure of financial wellbeing is wealth, and financial statements can be used to measure and manage wealth over the financial life cycle.
E2. Consumption is transferred forward and backward over time using payment series with compound interest. Understanding payment series requires applying the mathematical concepts of exponents and geometric series.
E3. Risk can be measured using probability and expected utility. This provides the means to evaluate risk management tools such as insurance.

[1] https://ficycle.org/math-research/#courseresultspaper

E4. Investments, including stocks, can be modelled as probability distributions. These distributions can be used to calculate the risk of investments.

These essential understandings require grasping a range of mathematical concepts and relating them to the relevant financial principles. Therefore, best practices require integrating financial education with mathematics education. We outline in more detail why conceptual understanding in finance improves learning and why this requires mathematics in our paper "Financial Education and Conceptual Understanding: Learning from Best Practices in Mathematics".[2]

Financial Outcomes

Our own research also finds that combining finance education with mathematics has beneficial effects upon financial *outcomes*. Previous research has shown that financial education has tended to have more significant effects on some types of outcome than others: generally, it's done better at getting people to engage in positive behavior (such as creating savings accounts) and worse at getting them to avoid negative behavior (such as incurring credit card fees).

We looked at how math factored into this and found that it significantly improved the effectiveness of financial education. As figure 1 below shows, respondents with low math confidence who took financial education were more likely to engage in positive behavior but also more likely to engage in negative behavior, while those with high math who take financial education are even more likely to engage in positive behavior but *less* likely to engage in negative behavior. In other words, the combined effects of math and finance appear to be complementary. We write about these results in detail in our paper "Positive and Negative Behavior: Differential Responses to Education".[3]

[2] https://ficycle.org/additional-education-research/#conceptualunderstanding
[3] https://ficycle.org/additional-education-research/#positivenegativebehavior

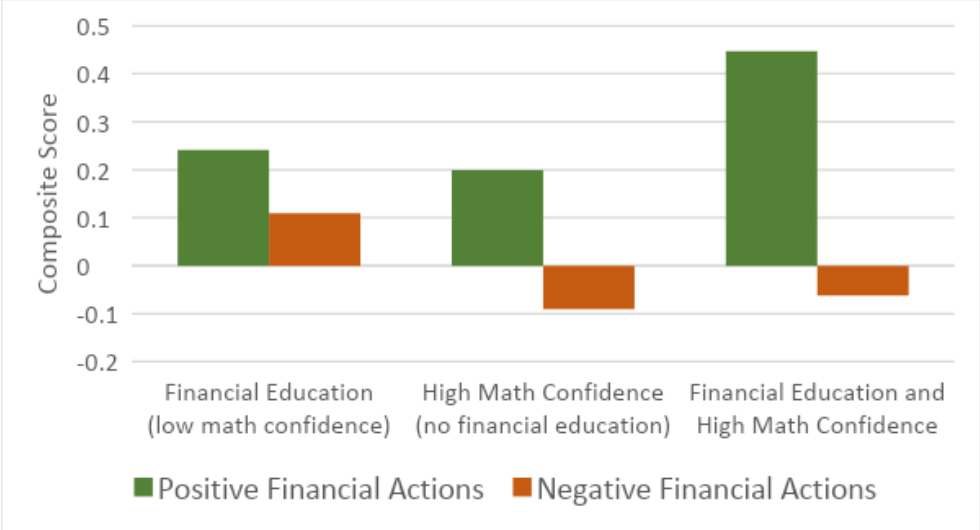

Figure 1

In addition, we look at the relationship between math and finance education and financial *confidence*. Research has shown that both over-confidence and under-confidence in one's financial abilities can lead to adverse financial outcomes. We find that over-confidence increases positive behavior a little, but significantly increases negative behavior, while under-confidence decreases positive behavior and increases negative behavior. Our research suggests that increasing math confidence and taking financial education both decrease financial under-confidence, but do not affect over-confidence. This suggests that the two measures increase confidence accuracy overall, but work remains in figuring out how they can better target over-confidence in particular. We discuss this in more detail in our paper "Calibrating Financial Confidence: The Role of Finance Education and Mathematical Confidence".[4]

Making Mathematics Relevant

Math achievement in the US is inadequate, especially at the high school level. US students consistently rank poorly compared to those in other developed countries. Further, research demonstrates that there is a strong link between mathematics achievements and economic outcomes. In particular, high levels of mathematics education are associated with high income levels.

A significant problem in education is student disengagement. In math class in particular, it's often reported that students are uninterested in the materials and don't view it as related to 'real life'. It has been shown, however, that learning is much more effective when students are highly engaged. One way to improve engagement is through attention to the material being taught. Studies show improved results

[4] https://ficycle.org/additional-education-research/#financialconfidence

when students find the material relevant and interesting (Greene et al 2004; Miller, DeBacker & Greene 1999).

Mathematics leaves much room for improvement in this regard. Many so-called real-world applications of math are contrived word problems, where supposed application is irrelevant to the solution of the problem. What's needed are authentic applications, where the mathematical processes can actually be used to guide decision-making (Bonotto 2007; Gerofsky 1996).

We believe that applications to finance are well-suited for this purpose. High school level mathematics facilitates understanding many financial problems that are relevant to students, making for an authentic and interesting application.

To verify our conjecture, we created a survey to assess the level of student interest in the topic of personal finance, to see how engaging it would be for them. We gave a set of questions to 250 students participating in our program at the start of the 2019-2020 school (before they had taken the course). In this survey, this we provided a list of common applications of mathematics, including personal finance. We asked students to rate each subject on a scale of 1-5 both in terms of their interest in the subject matter, and how important they thought it was to learn about the subject. These questions assessed the intrinsic value and instrumental value of the subjects, respectively.

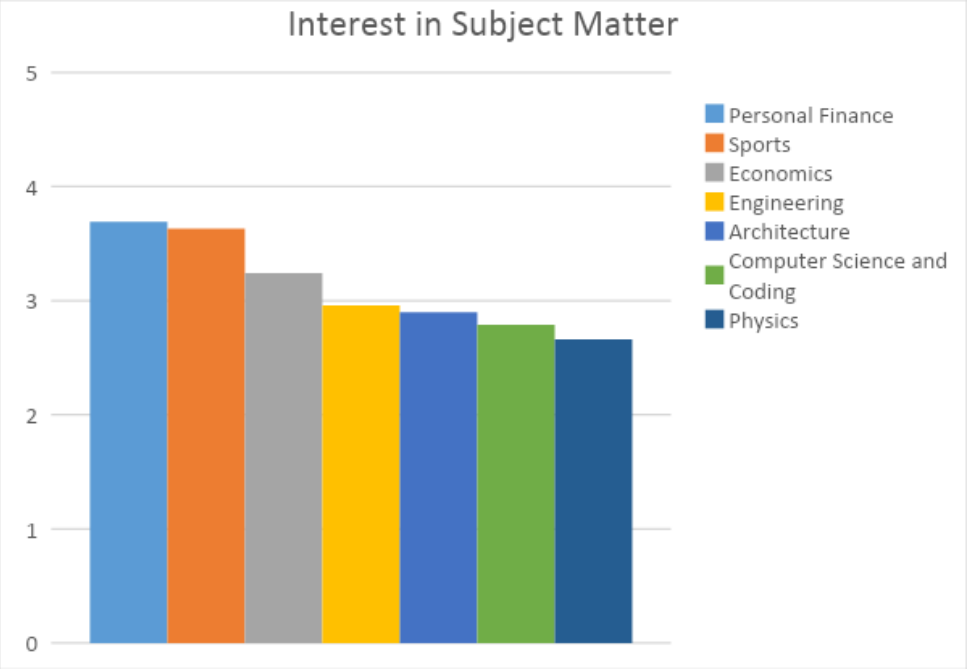

Figure 2

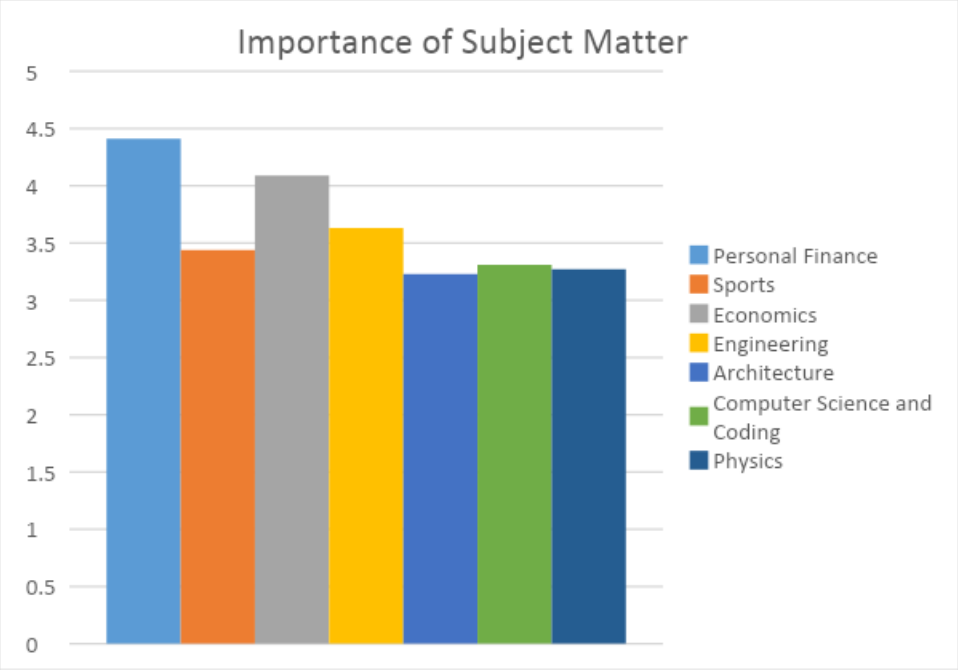

Figure 3

These results tell us a couple of interesting things. In line with our hypothesis, and validating our approach, students on average ranked personal finance highest, out of the given options, for both interest level and importance. In addition, it is worth noticing that the ranking of subjects below personal finance changed between the two questions. Most notably, sport was ranked a close second in terms of interest, but a distance fourth in terms of importance – and roughly the converse for economics. This demonstrates that students were distinguishing the question of interest and importance, emphasizing the significance of personal finance ranking first in both.

Taken together, this suggests that integrating financial applications into a mathematics course will improve mathematics education by increasing engagement. We discuss these results further in our paper "Enriching Mathematics through an Application to Finance".[5]

Opportunity Cost

Recall our fifth principle: "Student and teacher time is a scarce and valuable resource." There are only so many hours in the school year, both for students to learn and for teachers to teach, and all of these hours are already in use. Therefore, to add something to the curriculum requires removing something

[5] https://ficycle.org/additional-education-research/#mathengagement

else. To justify such an addition, it must be shown that the benefits of adding the new course outweigh the costs of removing the old one.

This is a prominent problem for alternative approaches to financial education that recommend implementing a standalone course in personal finance. To do this, some course in an unrelated subject matter must be eliminated – from the arts, sciences, or humanities. For example, a new financial education initiative in North Carolina required cutting the high school American history coursework in half in order to accommodate the personal finance requirement (Glenn 2019). Arguing that the benefits here clearly outweigh the costs is a tough case to make.

We also question the efficacy of the "no cost option" of making finance education an extra-curricular course. Though this appears to offer the benefits of financial education without any cost, since nothing has to be cut, the appearance is deceptive. First, students participating in one extra-curricular activity likely reduces their participation in another, and there are all many benefits to students participating in sports or creative arts, among other things. Second, an extra-curricular course is likely to be shorter and less rigorous than one taught within regular school hours. Research shows that less rigorous courses are less effective (Urban et al 2015).

Our approach, on the other hand, does not require eliminating a course, but instead changing the focus of one high school mathematics course such as a traditional Algebra 2 course. In this case, the opportunity cost arises from the change in focus of the mathematics education and the likelihood that the financial mathematics course covers fewer mathematical topics since time must also be set aside to discuss the financial concepts and applications.

We do not believe these costs are too severe, based on the previous discussion of best practices in mathematics education. Beyond mastering arithmetic, the significance of mathematics education lies primarily in developing mathematical *reasoning*, rather than getting students acquainted with as many mathematical concepts as possible: quality rather than quantity is our goal. Not, as current practice is often described, a curriculum that is "a mile wide and an inch deep" (National Governor's Association 2010 p. 3). If, as we argue, spending time exploring mathematical applications to finance increases students' engagement and fosters a deeper understanding of mathematical concepts, the tradeoff is justified.

Our Study

To test the validity of our principles, we have created a course in line with our standards and principles, and have conducted a preliminary study to assess its effectiveness directly. We created a curriculum for a

math course applied to finance that covers the topics we've outlined. A detailed description of the course structure is provided in "FiCycle Standards for Finance and Mathematics".[6]

We have piloted the course in a number of US high schools. To measure its effectiveness, we have assessed the knowledge and attitudes of program participants. For this, we instructed group of students taking a course in financial mathematics in the 2018-2019 school year to complete an assessment on both math and finance before and after the course, allowing us to track their progress. The responses were anonymous and did not count towards the student's grade. However, students input an ID number that could be used to match their beginning and end responses. You can find out more on the assessment tool in "FAME: The Finance and Mathematics Examination".[7]

The results showed that the students' knowledge increased significantly across all areas, over the course of the year, as summarized in the table below:

	Pre Assessment	Post Assessment	Change	SD	Change/SD
Total Score	44.08%	49.7%	5.65%***	12%	0.41
Finance Score	43.67%	47.99%	4.31%***	15%	0.28
Math Score	43.36%	51.85%	8.49%***	18%	0.45
Math Confidence	59.75%	62.17%	2.43%***	17%	0.14
Finance Confidence	56.12%	59.22%	3.11%***	14%	0.22

Table 1: Here and throughout: * significant at 10%; **significant at 5%; *** significant at 1%

We see that there is significant improvement across all categories of the assessment and, as a multiple of standard deviation, it is at or above the level of many successful education interventions (Lipsey & Wilson 1993).

In addition, we can check the relationship between the math and finance scores using regression analysis, where math score is the independent variable, and finance score is the dependent variable. The values are given in table 2.

[6] https://ficycle.org/math-research/#ficyclestandards
[7] https://ficycle.org/math-research/#famepaper

	Coefficient Value	R^2
Pre-Assessment	0.31***	0.14
Post-Assessment	0.51***	0.34

Table 2

As with previous research, there is a clear correlation between mathematical knowledge and financial knowledge. Also note that both the coefficient value and the degree of correlation R^2 are significantly higher in the post-assessment than in the pre-assessment.

To learn more, we also looked at how *individual's* scores changed over the course of the year. For this, we compared how an individual student's improvement levels across different categories were related. The primary result is a regression, with change in math score as the independent variable and change in finance score as the dependent variable. This gives us a model of the form $y = \alpha + \beta x$, where x is change in math score and y is change finance score. This tells us that for every percentage point improvement in math score a student makes from pre- to post-assessment, our model predicts that the student's finance score will improve by β percentage points.

In line with our hypothesis, there was a statistically significant relationship, with $\beta = 0.32$ and $R^2 = 0.15$, significant at 1%. In other words, on average, for every percentage point a student's math score increased from pre- to post- assessment, their finance score increase by approximately one-third of a percentage point. Further, approximately 15% of the improvement in finance score was explained by this model. The corresponding scatterplot and regression line are presented in *figure 4*:

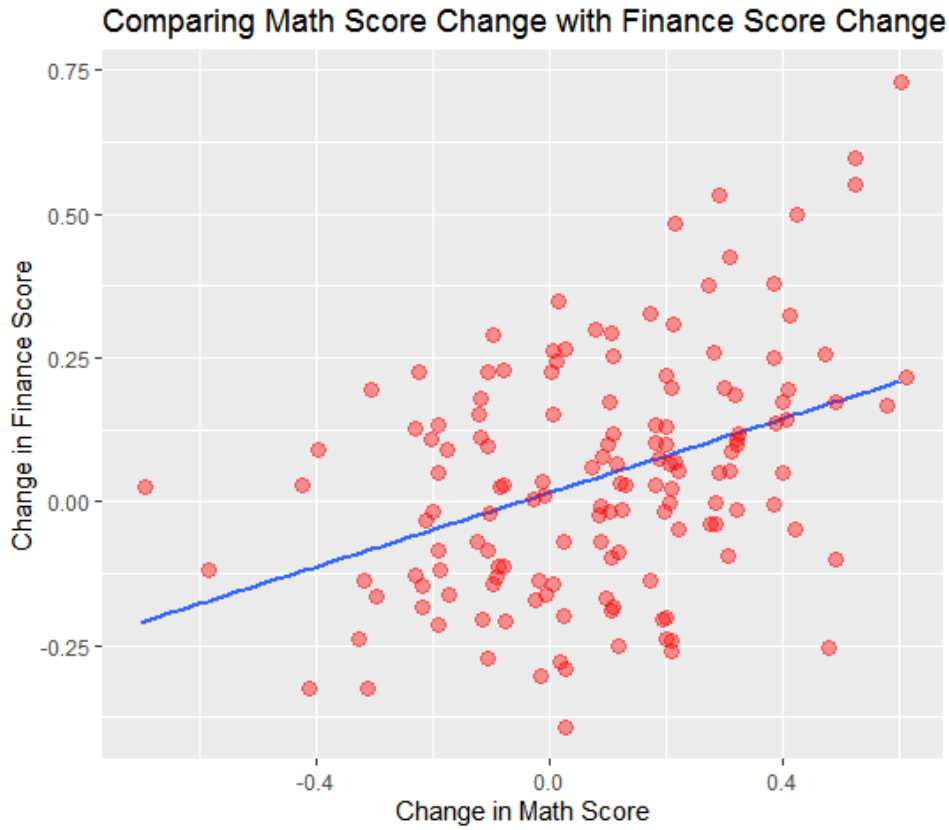

Figure 4

These results support our hypothesis that there is a causal relationship between mathematical learning and financial learning. Building on well-established results showing that mathematical knowledge and financial knowledge are correlated, we show that with an appropriate education intervention, improving mathematical knowledge is correlated with improving financial knowledge. This implies that the correlation between math and finance is not simply down to confounding factors that are contingently associated with a higher level of math education. You can find further details on this study in our paper "Best Practices in Financial Education: Incorporating Mathematics".[8]

Conclusion

Our results, along with previous research makes a strong case for incorporating finance education within a mathematics course. In addition to being supported by empirical findings, this approach is in line with the recommendations of influential education organizations like NCTM. Though much more work remains to be done, we believe that such courses have an important place in future high school

[8] https://ficycle.org/math-research/#courseresultspaper

curriculums. We hope to continue to refine our understanding of what makes for the most effective forms of financial education, and work to implement such courses in schools across the US.

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