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Enriching Mathematics through an Application to Finance Web Version

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Providing a high-quality mathematics education to young people in the US is of vital importance, and it is an area where there is a great deal of room for improvement. We suggest that one part of this process is a greater emphasis on applications of mathematics. At the high school level, a promising way to do this is by implementing an algebra course based upon applications to personal finance. This specific application has two key virtues. First, it is an authentic application that allows for sophisticated modeling activities that build conceptual understanding. Second, it is an application that is relevant to students, which builds motivation for engaging with the material.

1. Introduction

Providing a high-quality mathematics education to young people in the US is of vital importance, and it is an area where there is a great deal of room for improvement. We suggest that one part of this process is a greater emphasis on *applications* of mathematics. At the high school level, a promising way to do this is by implementing an algebra course based upon applications to personal finance.

This specific application has two key virtues. First, it is an authentic application that allows for sophisticated *modeling* activities that build conceptual understanding. Second, it is an application that is relevant to students, which builds motivation for engaging with the material. To make our case, we will begin by summarize research showing that both authentic applications of mathematics and relevant applications of mathematics improve student achievement. Following this, we will explain how a course applying high school algebra to personal finance meets these criteria.

2. A Strategy for Improving Mathematic Education

One promising area improving mathematics education is finding high quality *applications* of mathematics. The benefits here are twofold: first it allows for effective mathematical modeling; second it increases student motivation to engage with the material.

2.1 Mathematical Modeling

Developing skills in mathematical modeling is widely recognized to be an essential part of a high-quality mathematics education. It is a key point of emphasis in the Common Core State Standards for mathematics (National Governors Association 2010). In addition, skills relating to modeling are intertwined with virtually all of the other 'mathematical practices' that should underpin all mathematics education, according to the standards.

A similar emphasis on modeling is found in the recommended 'mathematics teaching practices' of the National Council of Teachers of Mathematics' (NCTM) influential manifesto, *Principles to Action*:

> An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world. (NCTM 2014 p. 70)

Why is modeling so important in mathematics education? It encourages the kind of reflection and 'deep strategy use' necessary to build conceptual understanding, which is essential for lasting mathematics success (Bullmaster-Day 2006; Brown et al 2014). Further, as a matter of definition, any occasion in which students go on to use mathematics outside of the classroom will involve modeling.

When modeling, students can relate the math they're learning to a tangible situation; this allows them to *reflect* on their calculations, in terms of what they say about the situation being modeled. It encourages in-depth project-based learning, as students develop the math to model more aspects of the scenario, and to engage in *experimentation* as they alter the parameters of the model and observe its effects. Finally, they may be able to draw on their background knowledge of the scenario to assist in tackling the problem, as recommended by constructivist theories of education: they are able to relate what they learn mathematically to the application, which builds conceptual understanding (Ellis 2018).

In order for these benefits to be realized, however, the model must be an *authentic application* of mathematics, not a mere 'word problem'. As Bonotto notes, addressing the latter kinds of problems tends to require a "suspension of sense-making, and rarely reaches the idea of mathematical modelling and mathematisation." (Bonotto 2007 p. 185)

To highlight how word problems in mathematics are deficient, Gerofsky (1996) subjects them to linguistic analysis and finds that experienced problem solvers learn to ignore real world implications of the scenario and avoid bringing in any background knowledge they might have. Thinking carefully about the particulars of the story not only does not help in solving the problem, it can actually hinder it! Consider the following example, also discussed by Gerofsky:

> Every year Stella rents a craft table at a local fun fair and sells the sweaters she has been making all year at home She has a deal for anyone who buys more than one sweater. She reduces the price of each additional sweater by 10% of the price of the previous sweater that the person bought Elizabeth bought *5* sweaters and paid $45.93 for the fifth sweater. How much did the first sweater cost? (Ebos, Klassen, & Zolis 1990)

In this example, one must extract an equation to solve from the paragraph of text; however, the scenario described makes no sense if one tries to analyze it. The unknown quantity is the price of the first sweater, despite the fact that the description stipulates that one of the known quantities (the price of the fifth sweaters) is determined by this unknown. There's no possible way you could be in this situation, so it's not practice for using math in real world retail – trying to put yourself in Stella's shoes would be no help in solving the problem, and could well lead to further confusion.

Such activities may be useful in developing linguistic reasoning, but they do not help with the higher-level mathematical abilities that are required for *modeling* in the true sense of the world (Masingla et al 1996). In addition, Ellis (2018) notes that authentic modeling task can be used to create what he calls "Culturally Responsive Teaching of Mathematics".

To summarize, effective use of mathematical modeling is one aspect of a high-quality mathematics education. This requires two key factors. First, the applications must be *authentic* rather than mere word problems, so that analyzing the scenario being modeled enhances the activity, and promotes learning. Second, the situation being modeled should be relevant to the students, so they can connect it with their existing knowledge and enhance their conceptual understanding.

We will show that applications to finance allow for exactly this kind of modeling. First, though, we will discuss another important element of improving math education: student engagement.

3.2 Student Engagement

One of the key factors in determining whether students succeed in mathematics is whether they are *engaged* with the subject matter. When students are engaged with material, they invest attention and effort in understanding topics and solving problems. Crucially, this is not about brute number of hours spent working, but about the *type* of activity the student performs: it must be the kind of high-level cognitive engagement which requires effort and attention. As Greene puts it, engagement requires meaningful, as opposed to shallow, cognitive processing: "Research has consistently found that meaningful processing strategies lead to greater performance on achievement measures over the material studied than shallow strategies." (Greene 2004 p. 463)

Therefore, in order for students to effectively learn mathematics, they must engage in high-level cognitive processing with the subject matter. This suggests that we should identify what factors promote these 'meaningful processing strategies'. On this Greene writes: "Three motivational factors that have been consistently related to cognitive strategy use in learning situations are self-efficacy… achievement goals… and perceived instrumentality" (Greene 2004 p 463).

The first two factors are somewhat removed from choice of application. 'Self efficacy' refers to the idea that students are more engaged when they have belief in their own ability to be successful in the task. 'Achievement goals' refers to what the student is aiming for when completing the task: they are more successful when they are concerned with mastering the subject matter, an 'achievement goal', than when they are concerned with getting a good grade, a 'performance goal'.

The factor that can be most directly affected by choice of application is the third: 'perceived instrumentality', which Greene (2004) describes as "the extent to which school tasks are perceived as instrumental to attaining personally valued future goals". For a subject or topic to have high instrumental value for a student, they must believe that developing skills and knowledge in that area will allow them to meet important goals down the line.

Why is instrumentality important? First, based on a theoretical account of human decision-making: expected value theory. In order to decide to engage with the study material, students need to believe putting in the effort will be worth their while. Therefore, the student must decide whether the benefits they will achieve if they are successful in learning, weighted according to how likely they are to succeed, outweigh the costs associated with making the effort. If this is so, they will decide to try; if not, they will not (Wigfield, 1994).

Of course, this is an idealized model of how a completely rational student would decide how hard to work, which is not a perfect match with reality. As Pintrinch and De Groot (1990) note, there is an emotional component to a student's willingness to engage.

Despite this, there is strong evidence that increasing perceived instrumentality increases student engagement – even if they don't always act in perfect accordance with expected value theory. As Wigfield (1994) notes: "Students' valuing of mathematics most strongly predicts their intentions to continue taking mathematics courses and their actual decisions to enroll in advanced math during high school." (p. 54) Similarly, Miller (1999) provides empirical results backing up this connection. Further, increasing perceived instrumentality of mathematics is especially important at the high school level, as this is the time engagement tends to decline, as Wigfield (1994) notes.

Changing the subject matter of high school mathematics to topics a student values is a promising way to reverse this trend of declining engagement. Though the focus here is only on one of three factors promoting engagement, it's plausible to think these factors are mutually reinforcing. If the student has future goals they are focused on, then what counts for them is mastery rather than performance. In addition, if the initial motivation leads the students to increased success, that will in turn increase self-efficacy.

4. The Value of Personal Finance

Basing a high-school math course around applications to personal finance will allow for high-quality modeling activities and increase the perceived instrumentality of math for many students.

4.1 Modeling Finance with Mathematics

To understand the connection between these subjects, it's necessary to take a step back and think about the nature of personal finance itself. The central topics in this area build upon two key principles: *transferring wealth across time*; and *managing risk*.

This is a result of Modigliani's *Life Cycle Hypothesis* (Ando & Modigliani 1963). In order to meet our needs at all points in our life, we need to transfer wealth acquired in periods when income is high (i.e., the peak of one's career) to periods when income is low (i.e. as a student and after retirement). Financial tools allow us to do this. Crucially, transferring wealth almost always involves *interest*: you pay it when you borrow, and you earn it when you invest. With interest comes the mathematics of exponents, logarithms and geometric series. Modeling, for example, retirement savings provides a complex and authentic application of these mathematical topics.

Turning to the second principle, at various points in our lives we will face financial risks: for example, the possibility we will need an expensive medical procedure or that our home will be destroyed in a fire. If the risk is higher than we can tolerate, we can use financial tools to mitigate it – we can pay someone else to take on the risk for us, by purchasing insurance. In addition, if we are able to take on additional financial risk, we can receive payment for doing so – for example by investing in the stock market.

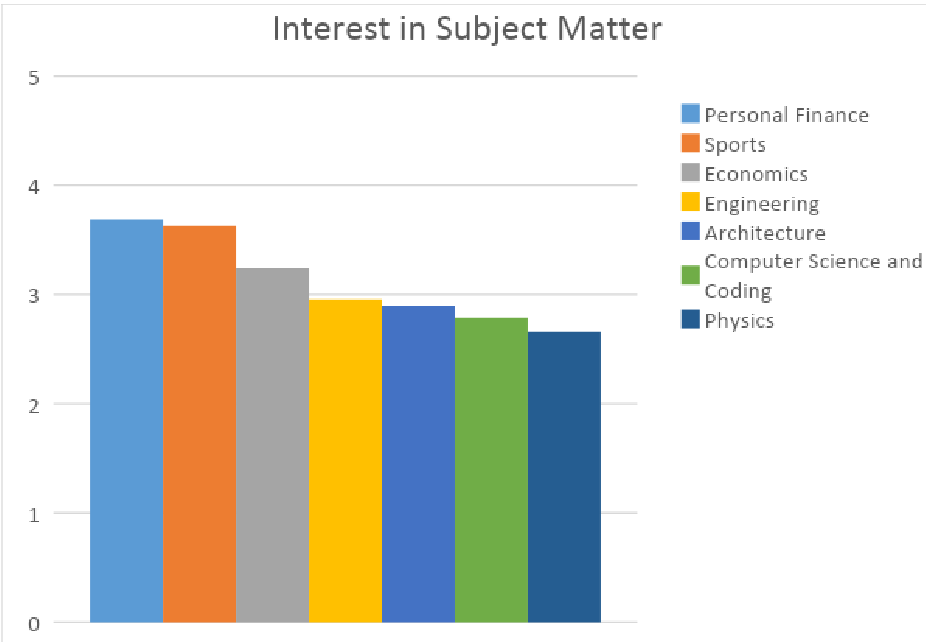
As a matter of definition, any scenario concerning risk will involve probability. Further, when assessing investment decisions, one must draw upon expected value and statistical topics, such as normal distributions. Again, this provides ground for high-quality, authentic modeling exercises.

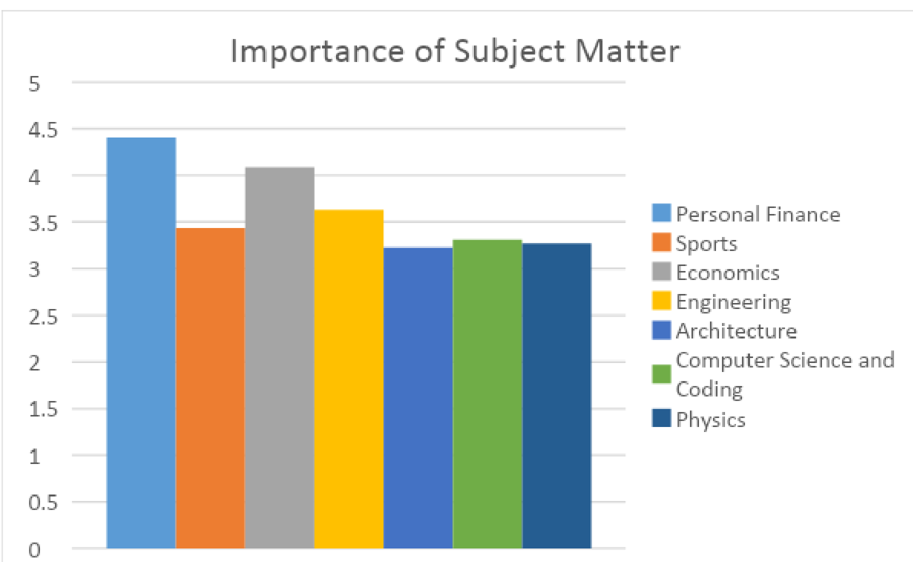
4.2 The Perceived Instrumentality of Finance

The reason why applying math to personal finance increases perceived instrumentality is intuitively clear. Students will need to make good financial decisions throughout their lives and improving their financial knowledge will help them do so. This is where using authentic applications, as discussed above, is essential – the math they are learning really will help with financial problems they are likely to face in the future.

In addition, research shows that current high-school students are in fact concerned about their financial future – this is a goal they will personally endorse (H&R Block 2016; NPR 2017; Ohio State News 2015). Given that this is the case, there is good reason to think that focusing on financial applications of math will increase perceived instrumentality which will in turn increase student engagement with the subject matter. Based on this, we created a study to confirm this directly.

We gave a set of questions to 250 students at the start of the 2019-2020 school year. In this survey, this we provided a list of common applications of mathematics, including personal finance. We asked students to rate each subject on a scale of 1-5 both in terms of their interest in the subject matter, and how important they thought it was to learn about the subject. These questions assessed the intrinsic value and instrumental value of the subjects, respectively. The results are displayed the graphs below.





These results tell us a couple of interesting things. In line with our conjecture, students on average ranked personal finance highest, out of the given options, for both interest level and importance. In addition, it is worth noticing that the ranking of subjects below personal finance changed between the two questions. Most notably, sport was ranked a close second in terms of interest, but a distance fourth in terms of importance – and roughly the converse for economics. This demonstrates that students were distinguishing the question of interest and importance, emphasizing the significance of personal finance ranking first in both.

Taken together, this suggests that integrating financial applications into a mathematics course will improve mathematics education by increasing engagement.

5. Conclusion

We have seen first that mathematics learning can be improved through authentic and relevant applications, and second that applications of algebra to finance are authentic and relevant for high school students. This suggest that a course on the mathematics of personal finance could improve mathematics achievement in the US. Given the urgent need for improvement here, this is an avenue well worth pursuing.

References

Ando, A., & Modigliani, F. (1963). The" life cycle" hypothesis of saving: Aggregate implications and tests. *The American economic review*, *53*(1), 55-84.

Bonotto, C. (2007). How to replace word problems with activities of realistic mathematical modelling. In *Modelling and applications in mathematics education* (pp. 185-192). Springer, Boston, MA.

Ebos, F., Klassen, J. & Zolis, P (1990) *Math matters: Book* 3 *(BritishColumbia edition).* Scarborough, Ont: Nelson Canada

Ellis, D. (2018) Culturally Responsive Mathematics Teaching

Gerofsky, S. (1996). A linguistic and narrative view of word problems in mathematics education. *For the Learning of Mathematics*, *16*, 36-45.

Greene, B. A., Miller, R. B., Crowson, H. M., Duke, B. L., & Akey, K. L. (2004). Predicting high school students' cognitive engagement and achievement: Contributions of classroom perceptions and motivation. *Contemporary educational psychology*, *29*(4), 462-482.

H&R Block (2016) H&R Block Survey. Retrieved from https://investors.hrblock.com/news-releases/news-release-details/hr-block-survey-says-teens-would-give-smartphones-graduate

Husman, J., & Lens, W. (1999). The role of the future in student motivation. *Educational psychologist*, *34*(2), 113-125.

Masingila, J. O., Davidenko, S., & Prus-Wisniowska, E. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational studies in mathematics*, *31*(1-2), 175-200.

Miller, R. B., DeBacker, T. K., & Greene, B. A. (1999). Perceived instrumentality and academics: The link to task valuing. *Journal of Instructional Psychology*, *26*(4), 250-250.

National Council of Teachers of Mathematics (2014) Principles to Actions: Ensuring Mathematical Success for All. *NCTM, National Council of Teachers of Mathematics.*

National Governors Association. (2010). Common core state standards. *Washington, DC*.

NPR (2017). In The Weeks Before Freshman Year, Money Worries Aplenty. Retrieved from https://www.npr.org/sections/ed/2017/09/07/542574774/in-the-weeks-before-freshman-year-money-worries-aplenty

Ohio State News (2015). 70 percent of college students stressed about finances. Retrieved from https://news.osu.edu/70-percent-of-college-students-stressed-about-finances/

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of educational psychology*, *82*(1), 33.

Wigfield, A. (1994). Expectancy-value theory of achievement motivation: A developmental perspective. *Educational psychology review*, *6*(1), 49-78.