

Name:

Date:

Revisiting Prerequisite Knowledge for fractions of LOGS

1. List the first 8 square numbers:

1, 4, 9, 16, 25, 36, 49, 64

2. List the square root of the first 8 square numbers:

1, 2, 3, 4, 5, 6, 7, 8

3. What makes a number a square number?

A square number is the product of some integer and itself.

4. List the first 5 cubic numbers:

1, 8, 27, 64, 125

5. List the cubed root of the first 5 cubic numbers:

1, 2, 3, 4, 5

6. What makes a number a cubic number?

A cubic number is the product of three identical integers.

7. List the first 3 biquadratic numbers (Numbers that can be represented as a whole number to the fourth power, n^4):

1, 16, 81

8. List the fourth root of the first 3 biquadratic numbers:

1, 2, 3

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Fractions of LOGS

Required Materials: 3 x LOG 2, 2 x LOG 5, 2 x LOG 4, 2 x LOG 8, LOG 10, LOG 16, LOG 25, LOG 64, and LOG 100

Part I: Discovering Log Properties

1. What LOG is half the height of a LOG 25? **LOG 5**

This can be written this using the following notation: $\frac{1}{2} \bullet \text{LOG } 25 = \text{LOG } \underline{5}$

2. What LOG is half the height of a LOG 100?

This can be written this using the following notation: $\frac{1}{2} \bullet \text{LOG } 100 = \text{LOG } \underline{10}$

3. What LOG is half the height of a LOG 4?

$$\frac{1}{2} \bullet \text{LOG } 4 = \text{LOG } \underline{2}$$

4. What LOG is half the height of a LOG 64?

$$\frac{1}{2} \bullet \text{LOG } 64 = \text{LOG } \underline{8}$$

5. Describe (in symbols, words or both) how you can determine the argument:

$$\frac{1}{2} \bullet \log A = \log \underline{A^{1/2}} = \text{LOG } \underline{2\sqrt{A}}$$



Part II: Applying knowledge

We have multiple ways to think about and explain this result.

- 1) An example of multiple LOGS where the multiple happens to be a fraction:

We know that: $\mathbf{B \bullet \text{LOG } A = \text{LOG } A^B}$,

Applying this we get: $\frac{1}{2} \bullet \text{LOG } 25 = \text{LOG } 25^{\frac{1}{2}}$

- 2) We also know from lining up and looking at our logs that:

Two LOG 5s are the same height as a LOG 25.
(Or that a LOG 5 is half the height of a LOG 25.)

Putting these two ways of thinking about $\frac{1}{2} \cdot \text{LOG } 25$ together we can see that:

$$\frac{1}{2} \cdot \text{LOG } 25 = \text{LOG } 25^{\frac{1}{2}} = \text{LOG } 5 \text{ and by extension } 25^{\frac{1}{2}} = \sqrt{25} = 5$$

Raising a quantity, a , to the $\frac{1}{2}$ power is the same as taking the square root of the quantity: $a^{\frac{1}{2}} = \sqrt{a}$

Examples: $25^{\frac{1}{2}} = \sqrt{25} = 5$ $100^{\frac{1}{2}} = \sqrt{100} = 10$ $9^{\frac{1}{2}} = \sqrt{9} = 3$

Part III: Generalizing

6. What LOG is half the height of a LOG 16?

This can be written this using the following notation: $\frac{1}{2} \cdot \text{LOG } 16 = \text{LOG } 16^{\frac{1}{2}} = \text{LOG } \underline{4}$

7. What LOG is one fourth the height of a LOG 16?

This can be written this using the following notation: $\frac{1}{4} \cdot \text{LOG } 16 = \text{LOG } 16^{1/4} = \text{LOG } \underline{4}$

8. What LOG is one third the height of a LOG 8?

$$\frac{1}{3} \cdot \text{LOG } 8 = \text{LOG } \underline{8^{1/3}} = \text{LOG } \underline{2}$$

9. What LOG is one third the height of a LOG 64?

$$\frac{1}{3} \cdot \text{LOG } 64 = \text{LOG } \underline{64^{1/3}} = \text{LOG } \underline{4}$$

10. What LOG is $\frac{1}{3}$ the height of a LOG A?

$$\frac{1}{3} \cdot \text{LOG } A = \text{LOG } A^{\frac{1}{3}} = \text{LOG } \underline{\sqrt[3]{A}}$$

How large is $\log A^{\frac{1}{3}}$? → It is $\frac{1}{3}$ the size of $\log A$.

How large is $\log \sqrt[3]{A}$? → It is $\frac{1}{3}$ the size of $\log A$.

11. What LOG is $\frac{1}{n}$ the height of a LOG A?

$$\frac{1}{n} \cdot \text{LOG } A = \text{LOG } A^{\frac{1}{n}} = \text{LOG } \underline{\sqrt[n]{A}}$$

How large is $\log A^{\frac{1}{n}}$? → It is $\frac{1}{n}$ the size of $\log A$.

How large is $\log \sqrt[n]{A}$? → It is $\frac{1}{n}$ the size of $\log A$.



Part IV: Practice & Application

12. What LOG is $\frac{1}{3}$ the height of a LOG 125?

$$\frac{1}{3} \cdot \text{LOG } 125 = \text{LOG } \underline{\sqrt[3]{125}} = \text{LOG } 5$$

13. What LOG is $\frac{1}{5}$ the height of a LOG 32?

$$\frac{1}{5} \cdot \text{LOG } 32 = \text{LOG } \underline{\sqrt[5]{32}} = \text{LOG } 2$$

14. What LOG is $\frac{1}{5}$ the height of a LOG 1024?

$$\frac{1}{5} \bullet \text{LOG } 1024 = \text{LOG } \sqrt[5]{1024} = \text{LOG } 4$$

15. What LOG is $\frac{1}{10}$ the height of a LOG 1024?

$$\frac{1}{10} \bullet \text{LOG } 1024 = \text{LOG } \sqrt[10]{1024} = \text{LOG } 2$$

16. What LOG is $\frac{1}{2}$ the height of a LOG 81?

$$\frac{1}{2} \bullet \text{LOG } 81 = \text{LOG } \sqrt[2]{81} = \text{LOG } 9$$

17. What LOG is $\frac{1}{4}$ the height of a LOG 81?

$$\frac{1}{4} \bullet \text{LOG } 81 = \text{LOG } \sqrt[4]{81} = \text{LOG } 3$$

18. What LOG is $\frac{1}{4}$ the height of a LOG 625?

$$\frac{1}{4} \bullet \text{LOG } 625 = \text{LOG } \sqrt[4]{625} = \text{LOG } 5$$

19. What LOG is 20% the height of a LOG 625?

$$.2 \bullet \text{LOG } 625 = \text{LOG } \sqrt[2]{625}$$

20. What LOG is $\frac{1}{\pi}$ the height of a LOG 9?

$$\frac{1}{\pi} \bullet \text{LOG } 9 = \text{LOG } \sqrt[\pi]{9}$$

21. What LOG is $\frac{1}{\odot}$ the height of a LOG \ominus ?

$$\frac{1}{\odot} \bullet \text{LOG } \ominus = \text{LOG } \sqrt[\odot]{\ominus}$$

Make up your own! Make up three that work out to a with a LOG whole number argument:

22. What LOG is $\frac{1}{8}$ the height of a LOG 2 ?

$$\frac{1}{8} \bullet \text{LOG } 2 = \text{LOG } \sqrt[8]{2}$$

23. What LOG is $\frac{1}{27}$ the height of a LOG 5 ?

$$\frac{1}{27} \bullet \text{LOG } 5 = \text{LOG } \sqrt[27]{5}$$

24. What LOG is $\frac{1}{3}$ the height of a LOG 73 ?

$$\frac{1}{3} \bullet \text{LOG } 73 = \text{LOG } \sqrt[3]{73}$$

(Example answers. Students can make up any three as long as they work out.)

Evaluate

25. $8^{\frac{1}{3}}$ 2

27. $125^{\frac{1}{3}}$ 5

26. $10000000^{\frac{1}{7}}$ 10

28. $3^{\frac{1}{4}}$ $\sqrt[4]{3}$