Name:
Date:

Solving Equations Using LOGs

Part I: Discovering LOG Properties

1. $\text{Log}_2(2^4) =$ *4*
2. $\text{Log}_2(2^x) =$ *x*
3. $\text{Log}_a(a^b) =$ *b*

4. If $\text{Log}_2 8 = x$ then what is the value of x? *x = 3*

   Consider the equation: $2^x = 8$. How can you solve for the value of x?

   Just like you are allowed to add, subtract, multiply or divide both sides of an equation by the same number and maintain equality, you can also take the Log of both sides! What happens if we take the Log_2 of both sides?

$$2^x = 8$$
$$\text{Log}_2(2^x) = \text{Log}_2(8)$$
$$x = \text{Log}_2 8$$
$$x = 3$$

5. Take the Log_2 of both sides to solve for x: $2^x = 64$

   $Log_2(2^x) = Log_2(64)$
   $x = 6$

6. Take the Log_2 of both sides to solve for x: $2^x = 1024$

   $Log_2(2^x) = Log_2(1024)$
   $x = 10$

By taking Log of both sides of an equation we can re-write exponential expressions in terms of Logs!

Examples

$16 = 2^4$	→	$\text{Log}_2(8) = \text{Log}_2(2^4)$	→	$\text{Log}_2 8 = 4$
$32 = 2^5$	→	$\text{Log}_2(32) = \text{Log}_2(2^5)$	→	$\text{Log}_2 32 = 5$
$25 = 5^2$	→	$\text{Log}_5(25) = \text{Log}_5(5^2)$	→	$\text{Log}_5 25 = 2$
$1000 = 10^3$	→	$\text{Log}_{10}(1000) = \text{Log}_{10}(10^3)$	→	$\text{Log}_{10} 1000 = 3$

The patter generally

$$[Argument] = [Base]^{[Exponent]} \quad \rightarrow \quad \text{Log}_{[Base]}[Argument] = [Exponent]$$

Before moving on to Part II make sure everyone in your group has the same answers to the above problems and understands the pattern.

Part II: Applying knowledge. Fill in the table.

	Express Using Exponents (in Exponential Form)	*Express Using Logs (in Logarithmic Form)*
7.	$125 = 5^3$	$Log_5(125) = 3$
8.	$128 = 2^7$	$Log_2(128) = 7$
9.	$10000 = 10^4$	$Log_{10}(1000) = 4$
10.	$16 = 4^2$	$\text{Log}_4 16 = 2$
11.	$81 = 3^4$	$\text{Log}_3 81 = 4$
12.	$100 = 10^2$	$\text{Log}_{10} 100 = 2$

Part III: Applying knowledge. Solve each equation by taking the Log of both sides of the equation. Show each step, even if you can determine x in your head.

13. $9 = 3^x$

$Log_3(9) = Log_3(3^x)$

$3 = x$

14. $4^x = 64$

$Log_4(4^x) = Log_4(64)$

$x = 3$

15. $3^x = 27$

$Log_3(3^x) = Log_3(27)$

$x = 3$

16. $64 = 8^x$

$Log_8(64) = Log_8(8^x)$

$2 = x$

17. $10^x = 100000$

$Log_{10}(10^x) = Log_{10}(100000)$

$x = 3$

18. $7^{x+1} = 49$

$Log_7(7^{x+1}) = Log_7(49)$

$x+1 = 2$

$x = 1$

19. $2^{2x+1} = 16$

$Log_2(2^{2x+1}) = Log_2(16)$

$2x+1 = 4$

$2x = 3$

$x = 1.5$

20. $12^{x-1} = 144$

$Log_{12}(12^{x-1}) = Log_{12}(144)$

$x-1 = 2$

$x = 3$

21. $2^{x+1} = 2^5$

$Log_2(2^{x+1}) = Log_2(2^5)$
$x+1 = 5$
$x = 4$

22. $2^{2x+1} = 4^{10}$

$Log_2(2^{2x+1}) = Log_2(2^{20})$
$2x+1 = 20$
$2x = 19$
$x = 9.5$

23. $5^{4x-3} = 1$

$Log_5(5^{4x-3}) = Log_5 1$
$4x-3 = 0$
$4x = 3$
$x = .75$

24. $4^{-3x} \cdot 4^x = 16^5$

$4^{-3+x} = (4^2)^5$
$Log_4(4^{-2x}) = Log_4(4^{10})$
$-2x = 10$
$x = -5$

Before moving on to Part IV make sure everyone in your group understands and has the same answers to the problems in Part III.

Part IV: Practice & Application. Applying knowledge. Solve each equation by taking the Log of both sides of the equation. You may need to use a calculator. If necessary, round to the nearest hundredth.

25. $3^x = 100$

$Log_3(3^x) = Log_3(100)$
$x = 4.19$

26. $12^x = 20736$

$Log_{12}(12^x) = Log_{12}(20736)$
$x = 4$

27. $8^x = 405$

$Log_8(8^x) = Log_8(405)$
$x = 2.89$

28. $12^x = 200$

$Log_{12}(12^x) = Log_{12}(200)$
$x = 2.1322$

29. $20^x = 5530$

$Log_{20}(20^x) = Log_{20}(5530)$
$x = 2.88$

30. $4^x = 1250$

$Log_4(4^x) = Log_4(1250)$
$x = 5.14$

31. $15^x = 3375$

$Log_{15}(15^x) = Log_{15}(3375)$
$x = 3$

32. $2^x = 65536$

$Log_2(2^x) = Log_2(65536)$
$x = 16$

Part V: More Challenging Questions. In order to take the Log of both sides of an equation you must isolate the exponential expression first. If necessary, round to the nearest hundredth.

33. $3^x + 1 = 10$

$3^x = 9$
$Log_3(3^x) = Log_3(3375)$
$x = 3$

34. $12^x - 4 = 140$

$12^x = 144$
$Log_{12}(12^x) = Log_{12}(144)$
$x = 2$

35. $2(5^x) = 250$

$5^x = 125$

$Log_5(5^x) = Log_5(125)$

$x = 3$

36. $7(9^x) - 15 = 45912$

$7(9^x) = 45927$

$9^x = 6561$

$Log_9(9^x) = Log_9(6561)$

$x = 4$

37. $\frac{10^x}{4} = 250$

$10^x = 1000$

$Log_{10}(10^x) = Log_{10}(1000)$

$x = 3$

38. $\frac{7^x+1}{2} = 25$

$7^x + 1 = 50$

$7^x = 49$

$Log_7(7^x) = Log_7(49)$

$x = 2$

39. $\frac{11^x-1}{4} = 30$

$11^x - 1 = 120$

$11^x = 121$

$Log_{11}(11^x) = Log_{11}(121)$

$x = 1.96$

40. $4(6^x - 3) = 31092$

$6^x - 3 = 7773$

$6^x = 7776$

$Log_6(6^x) = Log_6(7776)$

$x = 5$

41. $7(11^x) = 777$

$11^x = 111$

$Log_{11}(11^x) = Log_{11}(111)$

$x = 1.96$

42. $\frac{2.5^x-2}{5} = 72.2$

$2.5^x - 2 = 361$

$2.5^x = 363$

$Log_{2.5}(2.5^x) = Log_{2.5}(3363)$

$x = 6.43$

43. $\frac{10^x}{3} = 3$

$10^x = 9$

$Log_{10}(10^x) = Log_{10}(9)$

$x = .95$

44. $\frac{2^x+6}{7} = 1$

$2^x + 6 = 7$

$2^x = 1$

$Log_2(2^x) = Log_2(1)$

$x = 0$

45. $10(1.04)^x = 20$

$1.04^x = 2$

$Log_{1.04}(1.04^x) = Log_{1.04}(2)$

$x = 17.67$

46. $75(1.03)^x = 225$

$1.03^x = 3$

$Log_{1.03}(1.03^x) = Log_{1.03}(3)$

$x = 37.17$

47. $11^{x+3} = 2233$

$Log_{11}(11^{x+3}) = Log_{11}(2233)$

$x + 3 = 3.22$

$x = .22$

48. $2^{7x-6} = 200$

$Log_2(2^{7x-6}) = Log_2(200)$

$7x - 6 = 7.64$

$7x = 13.64$

$x = 1.95$