

Name:

Date:

Do Now

	Number ( $n$ )	in Inches	in Centimeters	$\frac{\text{Log}_{10} n}{\text{Log}_2 n}$
1.	10	$\text{LOG}_{10} 10 =$	$\text{LOG}_2 10 =$	
2.	100			
3.	2			
4.	4			



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## Change of Base

Required Materials: LOG 2, LOG 4, LOG 10, LOG 20, LOG 100, a  $\text{Log}_2$  Ruler, and a  $\text{Log}_{10}$  Ruler.

### Part I: Discovering LOG Properties

1. Using your  $\text{Log}_{10}$  Ruler calculate  $\text{Log}_{10} 10$  and record your answer in the table.
2. Using your  $\text{Log}_2$  Ruler, calculate  $\text{Log}_2 10$  and record your answer in the table.
3. Complete the table below for the remaining numbers:

	Number ( $n$ )	$\text{Log}_{10} n$	$\text{Log}_2 n$	$\frac{\text{Log}_{10} n}{\text{Log}_2 n}$
1.	10	$\text{LOG}_{10} 10 =$	$\text{LOG}_2 10 =$	
2.	100			
3.	2			
4.	4			
5.	20			
6.	32			

4. Are you surprised by the result in the far right column? If so, why? If not, why were you expecting that to happen?

Part II: Discovering LOG Properties

5. In the previous section you discovered that regardless of what the argument was,  $\frac{\text{Log}_{10} n}{\text{Log}_2 n}$  always equals the same thing. To be precise:

$$\frac{\text{Log}_{10} n}{\text{Log}_2 n} = 0.301029995663981195213738894724493026768189881462108541310 \dots$$

This means that regardless of the argument,  $\text{Log}_2 n$  and  $\text{Log}_{10} n$  will always be in ratio. Observe this again by filling out the table below (round to the nearest thousandth):

	Number ( $n$ )	$\text{Log}_2 n$	$.30102 \cdot \text{Log}_2 n$	$\text{Log}_{10} n$
1.	10	$\text{Log}_2 10 =$		$\text{Log}_{10} 10 =$
2.	100			
3.	2			
4.	4			
5.	20			
6.	32			

Part III: Generalizing

As further experimentation would show, regardless of the ruler, we use we can use this fact to be able to calculate the value of a LOG with any base, even when it’s not a number that “works well” such as  $\text{Log}_2 8$  or  $\text{Log}_5 25$ . We can use ratios to calculate the measure of a LOG on any ruler (using any base) in the following manner:

**Change of Base Formula:**

$$\text{Log}_b a = \frac{\text{Log}_c a}{\text{Log}_c b}$$

Note:  $c$  can be any base, as long as the same base is used for both numerator and the denominator.

6. As you learned last time, your calculator has a LOG button on it. Whenever you use the LOG button on a calculator or computer and it does not display a base it is representing a  $\text{Log}_{10}$ . Verify that the change of base formula works by using it in your calculator to evaluate a LOG you already know:

Using your  $\text{Log}_2$  ruler evaluate:  $\text{Log}_2 16 =$

Using the Change of Base Formula write this as a quotient of two logs:  $\text{Log}_2 16 = \frac{\text{Log} \underline{\hspace{1cm}}}{\text{Log} \underline{\hspace{1cm}}}$

Using the Change of Base Formula & your calculator evaluate:  $\text{Log}_2 16 =$

Part IV: Practice & Application: Use your calculator and the change of base formula to calculate the following. If necessary, round to the nearest hundredth.

7.  $\text{Log}_6 36 =$  \_\_\_\_\_

11.  $\text{Log}_{70} 1000 =$  \_\_\_\_\_

8.  $\text{Log}_2 100 =$  \_\_\_\_\_

12.  $\text{Log}_{2.7} 17 =$  \_\_\_\_\_

9.  $\text{Log}_3 100 =$  \_\_\_\_\_

13.  $\text{Log}_{3.14} \pi =$  \_\_\_\_\_

10.  $\text{Log}_7 100 =$  \_\_\_\_\_

14.  $\text{Log}_{0.5} 25 =$  \_\_\_\_\_