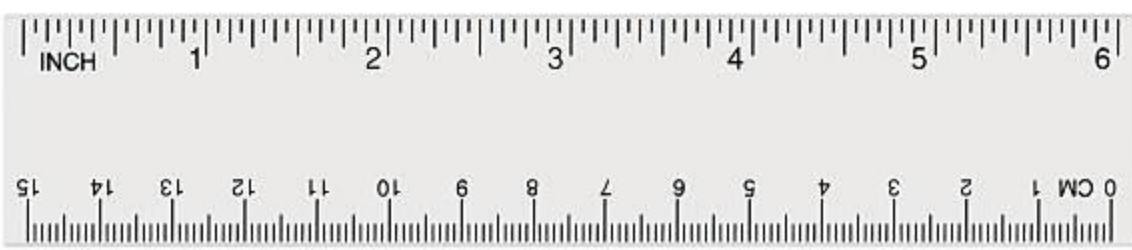
Name:
Date:

Do Now

	Number (n)	in Inches	in Centimeters	$\frac{\text{Log}_{10} n}{\text{Log}_2 n}$
1.	10	$\text{LOG}_{10}\, 10 =$ *3.2*	$\text{LOG}_2\, 10 =$ *7.8*	*.301*
2.	100	*9.1*	*15.4*	*.301*
3.	2	*1*	*2.6*	*.301*
4.	4	*1.9*	*5*	*.301*



Name:
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Change of Base

Required Materials: LOG 2, LOG 4, LOG 10, LOG 20, LOG 100, a Log_2 Ruler, and a Log_{10} Ruler.

Part I: Discovering LOG Properties

1. Using your Log_{10} Ruler calculate $\text{Log}_{10}10$ and record your answer in the table.

2. Using your Log_2 Ruler, calculate $\text{Log}_2 10$ and record your answer in the table.

3. Complete the table below for the remaining numbers:

	Number (n)	$\text{Log}_{10} n$	$\text{Log}_2 n$	$\frac{\text{Log}_{10} n}{\text{Log}_2 n}$
1.	10	$\text{LOG}_{10}\ 10 =$ *1*	$\text{LOG}_2\ 10 =$ *3.321*	*.301*
2.	100	*2*	*6.6439*	*.301*
3.	2	*.301*	*1*	*.301*
4.	4	*.602*	*2*	*.301*
5.	20	*1.301*	*4.3219*	*.301*
6.	32	*1.805*	*5*	*.301*

4. Are you surprised by the result in the far right column? If so, why? If not, why were you expecting that to happen?

(Reflective question for students; many possible responses. In general, most people should be surprised by the ratio being invariant)

Part II: Discovering LOG Properties

5. In the previous section you discovered that regardless of what the argument was, $\frac{\text{Log}_{10} n}{\text{Log}_2 n}$ always equals the same thing. To be precise:

$$\frac{\text{Log}_{10} n}{\text{Log}_2 n} = 0.30102999566398119521373889472449302676818988146210854131 0\ldots$$

This means that regardless of the argument, $\text{Log}_2 n$ and $\text{Log}_{10} n$ will always be in ratio. Observe this again by filling out the table below (round to the nearest thousandth):

	Number (n)	$\text{Log}_2 n$	$.30102 \cdot \text{Log}_2 n$	$\text{Log}_{10} n$
1.	10	$\text{Log}_2 10 =$ *3.321*	*1*	$\text{Log}_{10} 10 =$ *1*
2.	100	*6.644*	*2*	*2*
3.	2	*1*	*.301*	*.301*
4.	4	*2*	*.602*	*.602*
5.	20	*4.321*	*1.301*	*1.301*
6.	32	*5*	*1.151*	*1.151*

Part III: Generalizing

As further experimentation would show, regardless of the ruler, we use we can use this fact to be able to calculate the value of a LOG with any base, even when it's not a number that "works well" such as $\text{Log}_2 8$ or $\text{Log}_5 25$. We can use ratios to calculate the measure of a LOG on any ruler (using any base) in the following manner:

Change of Base Formula:

$$\text{Log}_b a = \frac{\text{Log}_c a}{\text{Log}_c b}$$

Note: *c* can be any base, as long as the same base is used for both numerator and the denominator.

6. As you learned last time, your calculator has a LOG button on it. Whenever you use the LOG button on a calculator or computer and it does not display a base it is representing a Log_{10}. Verify that the change of base formula works by using it in your calculator to evaluate a LOG you already know:

Using your Log_2 ruler evaluate: $\text{Log}_2 16 =$ *4*

Using the Change of Base Formula write this as a quotient of two logs: $\text{Log}_2 16 = \frac{\text{Log}\ \underline{16}}{\text{Log}\ \underline{2}}$

Using the Change of Base Formula & your calculator evaluate: $\text{Log}_2 16 =$ *4*

Part IV: Practice & Application: Use your calculator and the change of base formula to calculate the following. If necessary, round to the nearest hundredth.

7. $\text{Log}_6 36 =$ *2*

8. $\text{Log}_2 100 =$ *6.64*

9. $\text{Log}_3 100 =$ *4.19*

10. $\text{Log}_7 100 =$ *2.37*

11. $\text{Log}_{70} 1000 =$ *1.62*

12. $\text{Log}_{2.7} 17 =$ *2.85*

13. $\text{Log}_{3.14} \pi =$ *1*

14. $\text{Log}_{0.5} 25 =$ *-4.64*