Do Now

1. How long is your writing utensil in inches?
   
   (many possible answers)
   5.75 in

2. How long is your writing utensil in centimeters?
   
   (many possible answers)
   14 cm

3. Did you get the same answer to #1 and #2? How is it possible that the length of a writing utensil can be represented using two different numbers?

   No, I did not get the same answer to #1 and #2. This is because the same measurement can be represented with different units.

4. Can you give another example of how we can measure the same thing in two different ways and get different answers?

   (many possible answers)
   You can measure the weight of a rock using kilograms and pounds.
   You can measure the volume of lemonade using gallons and liters.
LOGS & Scale

Required Materials: LOG 2, LOG 4, LOG 8, LOG 10, LOG 16, LOG 32, LOG 40, LOG 64, and LOG 100

Part I: To right is what we call a “Log Base 2” ruler, typically written as “Log_2” ruler. We refer to the subscript (the small number) as the base. You use it just like a normal ruler: you line up the bottom of your LOG with the zero and measure the height by reading the number off the ruler.

1. What is the measure of a Log 2 on the Log_2 ruler?  
(Ignore the nub that connects LOGs when measuring them.)

This can be written this using the following notation: \( \log_2 2 = \_ \_ \_ \)

2. What is the measure of a Log 4 on the Log_2 ruler?  

This can be written this using the following notation: \( \log_2 4 = \_ \_ \_ \)

3. What is the measure of a Log 8 on the Log_2 ruler?  

This can be written this using the following notation: \( \log_2 8 = \_ \_ \_ \)

4. What is the measure of a Log 16 on the Log_2 ruler? \( \log_2 16 = \_ \_ \_ \)

5. What is the measure of a Log 32 on the Log_2 ruler? \( \log_2 32 = \_ \_ \_ \)

6. What is the measure of a Log 10 on the Log_2 ruler? \( \log_2 10 = \_ \_ \_ \_ \_ \_ \_ \)  
(It’s okay to estimate if you have to)

7. What is the measure of a Log 40 on the Log_2 ruler? \( \log_2 40 = \_ \_ \_ \_ \_ \_ \_ \)  
(It’s okay to estimate if you have to)

8. Which numbers measured out to whole numbers? Which numbers didn’t? What kinds of numbers are easiest to measure using a Log_2 ruler?  
Numbers that are powers of 2 measured out to be whole numbers. Numbers that are not powers of 2 didn’t. Powers of 2 are the easiest to measure using a Log_2 ruler.

9. Give an example of at least two additional numbers that would measure out to a whole number using a Log_2 ruler:  
(many possible answers)  
64, 128  

Give an example of at least two numbers that would NOT measure out to a whole number using a Log_2 ruler:  
(many possible answers)  
25, 73
Part II: To right is what we call a “Log Base 10” ruler, typically written as “Log_{10}” ruler. We refer to the subscript (the small number) as the base. You use it just like a normal ruler: you line up the bottom of your LOG with the zero and measure the height by reading the number off the ruler.

10. What is the measure of a Log 10 on the Log_{10} ruler? \[ \text{This can be written this using the following notation: } \log_{10} 10 = 1 \]

11. What is the measure of a Log 100 on the Log_{10} ruler? \[ \text{This can be written this using the following notation: } \log_{10} 100 = 2 \]

This can also be written as: \[ \log_{10} 10^2 = 2 \]

12. Even though we don’t have one, what would be the measure of LOG 1000 on the Log_{10} ruler? \[ \log_{10} 1000 = \log_{10} 10^3 = 3 \]

13. Even though we don’t have one, what would be the measure of LOG 10000 on the Log_{10} ruler? \[ \log_{10} 10000 = 4 \]

14. What is the measure of a Log 2 on the Log_{10} ruler? \[ \log_{10} 2 = \text{4} \]

(It’s okay to estimate if you have to)

15. What is the measure of a Log 16 on the Log_{10} ruler? \[ \log_{10} 16 = 1.2 \]

16. What is the measure of a Log 40 on the Log_{10} ruler? \[ \log_{10} 40 = 1.6 \]

17. Which numbers measured out to whole numbers? Which numbers didn’t? What kinds of numbers are easiest to measure using a Log_{10} ruler?

Numbers that are powers of 10 measured out to be whole numbers. Numbers that are not powers of 10 didn’t. Powers of 10 are the easiest to measure using a Log_{10} ruler.

18. Give an example of at least two additional numbers that would measure out to a whole number using a Log_{10} ruler:

(many possible answers)

100,000, 10,000,000

Give an example of at least two numbers that would NOT measure out to a whole number using a Log_{10} ruler:

(many possible answers)

27, 5,463
Part III: Generalizing

19. Arguments that are powers of ____ will measure out to be whole numbers on a Log₂ ruler.

20. Arguments that are powers of ____ will measure out to be whole numbers on a Log₁₀ ruler.

21. Arguments that are powers of ____ will measure out to be whole numbers on a Logₙ ruler.

22. Vocabulary:

23. Log₁₀ 10 = ____

24. Log₁₀ 10⁵ = ____

25. Log₂ 2⁵ = ____

26. Log₂ 2¹⁸ = ____

27. Log₁₀ 100000 = ____

28. Log₂ 128 = ____

29. Log₅ 5 = ____

30. Log₅ 5⁸ = ____

31. Log₅ 5⁴⁵⁸ = ____

32. Log₃ 5² = ____

33. Log₅ 2⁵ = ____

34. Log₅ 125 = ____

35. Log₃ 9 = ____

36. Log₃ 2⁷ = ____

37. Log₇ 4⁹ = ____

38. Log₄ 6⁴ = ____

39. Logₓ x = ____

40. Log₁₀ 10⁴.₅ = ____

41. Log₉ Q⁶ = ____

42. Log₂ 4³ = ____

43. Logₑ eʳᵗ = ____

44. Log₁₀ 10Log₁₀ 10 = ____

45. Describe in words how you would build or draw a Log₅ ruler if you were given a LOG 5 piece.

Mark each section of the ruler with integers from 0 to however long you want the ruler, break the ruler into equal segments, each the length of a LOG 5 piece. Label these segments 1, 2, 3, etc.