1. List the first 8 square numbers:
   \[1, 4, 9, 16, 25, 36, 49, 64\]

2. List the square root of the first 8 square numbers:
   \[1, 2, 3, 4, 5, 6, 7, 8\]

3. What makes a number a square number?
   
   A square number is the product of some integer and itself.

4. List the first 5 cubic numbers:
   \[1, 6, 27, 64, 125\]

5. List the cubed root of the first 5 cubic numbers:
   \[1, 2, 3, 4, 5\]

6. What makes a number a cubic number?
   
   A cubic number is the product of three identical integers.

7. List the first 3 biquadratic numbers (Numbers that can be represented as a whole number to the fourth power, \(n^4\)):
   \[1, 16, 81\]

8. List the fourth root of the first 3 biquadratic numbers:
   \[1, 2, 3\]
Fractions of LOGS

Required Materials: 3 x LOG 2, 2 x LOG 5, 2 x LOG 4, 2 x LOG 8, LOG 10, LOG 16, LOG 25, LOG 64, and LOG 100

Part I: Discovering Log Properties

1. What LOG is half the height of a LOG 25? \( \text{LOG } 5 \)

   This can be written this using the following notation: \( \frac{1}{2} \cdot \text{LOG } 25 = \text{LOG } 5 \)

2. What LOG is half the height of a LOG 100?

   This can be written this using the following notation: \( \frac{1}{2} \cdot \text{LOG } 100 = \text{LOG } 10 \)

3. What LOG is half the height of a LOG 4?

   \( \frac{1}{2} \cdot \text{LOG } 4 = \text{LOG } 2 \)

4. What LOG is half the height of a LOG 64?

   \( \frac{1}{2} \cdot \text{LOG } 64 = \text{LOG } 8 \)

5. Describe (in symbols, works or both) how you can determine the argument:

   \( \frac{1}{2} \cdot \log A = \log \sqrt[2]{A} \)

Part II: Applying knowledge

We have multiple ways to think about and explain this result.

1) An example of multiple LOGS where the multiple happens to be a fraction:

   We know that: \( B \cdot \text{LOG } A = \text{LOG } A^B \),

   Applying this we get: \( \frac{1}{2} \cdot \text{LOG } 25 = \text{LOG } 25^{\frac{1}{2}} \)

2) We also know from lining up and looking at our logs that:

   Two LOG 5s are the same height as a LOG 25. (Or that a LOG 5 is half the height of a LOG 25.)
Putting these two ways of thinking about $\frac{1}{2} \cdot \log 25$ together we can see that:

$$\frac{1}{2} \cdot \log 25 = \log 25^{\frac{1}{2}} = \log 5$$

and by extension $25^{\frac{1}{2}} = \sqrt{25} = 5$

Raising a quantity, $a$, to the $\frac{1}{2}$ power is the same as taking the square root of the quantity: $a^\frac{1}{2} = \sqrt{a}$

**Examples:**

- $25^{\frac{1}{2}} = \sqrt{25} = 5$
- $100^{\frac{1}{2}} = \sqrt{100} = 10$
- $9^{\frac{1}{2}} = \sqrt{9} = 3$

**Part III: Generalizing**

6. What $\log$ is half the height of a $\log 16$?

   *This can be written this using the following notation:* $\frac{1}{2} \cdot \log 16 = \log 16^{\frac{1}{2}} = \log 4$

7. What $\log$ is one fourth the height of a $\log 16$?

   *This can be written this using the following notation:* $\frac{1}{4} \cdot \log 16 = \log 16^{1/4} = \log 4$

8. What $\log$ is one third the height of a $\log 8$?

   $\frac{1}{3} \cdot \log 8 = \log 8^{\frac{1}{3}} = \log 2$

9. What $\log$ is one third the height of a $\log 64$?

   $\frac{1}{3} \cdot \log 64 = \log 64^{\frac{1}{3}} = \log 4$

10. What $\log$ is $\frac{1}{3}$ the height of a $\log A$?

    $\frac{1}{3} \cdot \log A = \log A^{\frac{1}{3}} = \log \sqrt[3]{A}$

    **How large is** $\log A^{\frac{1}{3}}$?  $

    \rightarrow$  *It is $\frac{1}{3}$ the size of* $\log A$.

    **How large is** $\log \sqrt[3]{A}$?  $

    \rightarrow$  *It is $\frac{1}{3}$ the size of* $\log A$.

11. What $\log$ is $\frac{1}{n}$ the height of a $\log A$?

    $\frac{1}{n} \cdot \log A = \log A^{\frac{1}{n}} = \log \sqrt[n]{A}$

    **How large is** $\log A^{\frac{1}{n}}$?  $

    \rightarrow$  *It is $\frac{1}{n}$ the size of* $\log A$.

    **How large is** $\log \sqrt[n]{A}$?  $

    \rightarrow$  *It is $\frac{1}{n}$ the size of* $\log A$.

**Part IV: Practice & Application**

12. What $\log$ is $\frac{1}{3}$ the height of a $\log 125$?

    $\frac{1}{3} \cdot \log 125 = \log 125^{\frac{1}{3}} = \log 5$

13. What $\log$ is $\frac{1}{5}$ the height of a $\log 32$?

    $\frac{1}{5} \cdot \log 32 = \log 32^{\frac{1}{5}} = \log 2$
14. What \( \log \) is \( \frac{1}{5} \) the height of a \( \log \) 1024? \[
\frac{1}{5} \cdot \log 1024 = \log \sqrt[5]{1024} = \log 4
\]

15. What \( \log \) is \( \frac{1}{10} \) the height of a \( \log \) 1024? \[
\frac{1}{10} \cdot \log 1024 = \log \sqrt[10]{1024} = \log 2
\]

16. What \( \log \) is \( \frac{1}{2} \) the height of a \( \log \) 81? \[
\frac{1}{2} \cdot \log 81 = \log \sqrt{81} = \log 9
\]

17. What \( \log \) is \( \frac{1}{4} \) the height of a \( \log \) 81? \[
\frac{1}{4} \cdot \log 81 = \log \sqrt[4]{81} = \log 3
\]

18. What \( \log \) is \( \frac{1}{4} \) the height of a \( \log \) 625? \[
\frac{1}{4} \cdot \log 625 = \log \sqrt[4]{625} = \log 5
\]

19. What \( \log \) is 20\% the height of a \( \log \) 625? \[
.2 \cdot \log 625 = \log \sqrt[5]{625}
\]

20. What \( \log \) is \( \frac{1}{\pi} \) the height of a \( \log \) 9? \[
\frac{1}{\pi} \cdot \log 9 = \log \sqrt[\pi]{9}
\]

21. What \( \log \) is \( \frac{1}{\odot} \) the height of a \( \log \) \( \odot \)? \[
\frac{1}{\odot} \cdot \log \odot = \log \sqrt[\odot]{\odot}
\]

Make up your own! Make up three that work out to a with a \( \log \) whole number argument:

22. What \( \log \) is \( \frac{1}{8} \) the height of a \( \log \) 2? \[
\frac{1}{8} \cdot \log 2 = \log \sqrt[8]{2}
\]

23. What \( \log \) is \( \frac{1}{27} \) the height of a \( \log \) 5? \[
\frac{1}{27} \cdot \log 5 = \log \sqrt[27]{5}
\]

24. What \( \log \) is \( \frac{1}{3} \) the height of a \( \log \) \( \sqrt[3]{3} \)? \[
\frac{1}{3} \cdot \log \sqrt[3]{3} = \log \sqrt[3]{3}
\]

(Example answers. Students can make up any three as long as they work out.)

Evaluate

25. \( 8^{\frac{1}{3}} \) \( \sqrt[3]{2} \) 2

26. \( 1000000^{\frac{1}{7}} \) 10

27. \( 125^{\frac{1}{3}} \) \( \sqrt[3]{5} \) 5

28. \( 3^{\frac{1}{3}} \) \( \sqrt[3]{3} \)